C30

Further trigonometry

CHAPTER ASSIGNMENT · 17 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Show your working and sketch the triangle. You may use a calculator. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise. Where a question refers to a diagram, the measurements are written out in words; look at the book page for the picture.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 30 (Further trigonometry), for classroom use. The printed sheet cannot carry the book's diagrams, so each question says what its diagram shows; the diagram is on the book page quoted in the section heading. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 30.1 · The sine rule (book p. 419)

30.1.1 Calculate the length of the side marked x in each of the following. (Sides and angles are named as the diagram shows them: "opposite" means the side facing that angle.)

(a) A triangle with angles 60° and 40°. The side of 12 cm is opposite the 60° angle. The side x cm is opposite the 40° angle.

(b) A triangle with angles 20° and 130°. The side of 20 cm is opposite the 130° angle. The side x cm is opposite the 20° angle.

(c) A triangle with angles 25° and 35°. The side of 9 mm lies between those two angles, so it is opposite the third angle. The side x mm is opposite the 35° angle.

(d) A four-sided figure with one diagonal drawn. The diagonal splits it into two triangles. The upper triangle has an angle of 30° and a side of 3 cm. The lower triangle has angles of 80° and 40° and a side x cm. The two short marks on the figure show two sides of equal length (the shared diagonal links the triangles).

30.1.2 Calculate the size of the angle marked θ° in each of the following.

(a) A triangle with an angle of 20°. The side of 5 cm is opposite the 20° angle. The side of 8 cm is opposite the angle θ°.

(b) A triangle with an angle of 30°. The side of 22 mm is opposite the 30° angle. The side of 35 mm is opposite the angle θ°.

(c) A triangle with an angle of 60°. The side of 8 cm is opposite the 60° angle. The side of 9 cm is opposite the angle θ°.

(d) A triangle with a line drawn from one corner to the opposite side, meeting it at a right angle. At the bottom-left corner the line makes angles of 30° and 40°. The sides of 6 cm and 7 cm and the angle θ° are marked as in the book diagram.

30.1.3 ΔABC has the following dimensions: AC = 10 cm, AB = 8 cm and angle ACB = 20°.

(a) Calculate the two possible values for angle CBA.

(b) Sketch and label the two possible shapes for ΔABC.

30.1.4 ΔPQR has the following dimensions: PQ = 6 cm, PR = 4 cm and angle PQR = 40°.

(a) Calculate the two possible values for angle QRP.

(b) Sketch and label the two possible shapes for ΔPQR.

Exercise 30.2 · The cosine rule (book pp. 421 to 422)

30.2.1 Calculate the length of the side marked x in each of the following.

(a) A triangle with sides of 2 m and 3 m that enclose an angle of 140°. The side x m is opposite the 140° angle.

(b) A triangle with sides of 6 cm and 10 cm that enclose an angle of 95°. The side x cm is opposite the 95° angle.

(c) A triangle with sides of 7 cm and 15 cm that enclose an angle of 25°. The side x cm is the third side.

(d) A triangle with two sides of 4 cm (the marks on the figure show they are equal) that enclose an angle of 45°. The side x cm is the third side.

(e) A triangle with sides of 5 cm and 7 cm that enclose an angle of 125°. The side x cm is opposite the 125° angle.

30.2.2 Calculate the angle marked θ° in each of the following.

(a) A triangle with sides of 20 m, 25 m and 40 m. The angle θ° is between the 20 m and 25 m sides, opposite the 40 m side.

(b) A triangle with sides of 2 mm, 4 mm and 5 mm. The angle θ° is between the 2 mm and 4 mm sides, opposite the 5 mm side.

(c) A triangle with sides of 9 cm, 9 cm and 15 cm (the marks on the figure show the two 9 cm sides are equal). The angle θ° is between a 9 cm side and the 15 cm side.

(d) A triangle with sides of 4 cm, 15 cm and 18 cm. The angle θ° is between the 4 cm and 18 cm sides, opposite the 15 cm side.

(e) A triangle with sides of 7 cm, 10 cm and 15 cm. The angle θ° is between the 7 cm and 10 cm sides, opposite the 15 cm side.

Exercise 30.3 · Sine rule and cosine rule problems (book p. 422)

30.3.1 Four players, W, X, Y and Z, are on a rugby pitch. The diagram shows a plan view of their relative positions. WX = 20 m, XY = 24 m, YZ = 40 m, ZW = 30 m, angle XYZ = 80°, and the line XZ is drawn across the quadrilateral WXYZ. Calculate:

(a) the distance between players X and Z,

(b) angle ZWX,

(c) angle WZX,

(d) angle YZX,

(e) the distance between players W and Y.

30.3.2 Three yachts, A, B and C, are racing off Cape Comorin in India. Their relative positions are shown in the diagram: AB = 220 m, AC = 450 m and angle BAC = 55°. Calculate the distance between B and C to the nearest 10 m.

30.3.3 There are two trees standing on one side of a river bank. On the opposite side, a boy is standing at X. Using the information given, calculate the distance between the two trees. (In the diagram the boy at X looks across the river at the two trees. The first tree is 40 m from X, the second tree is 50 m from X, and the angle between the two lines of sight is 110°.)

Exercise 30.4 · Area of a triangle and the shortest distance from a point to a line (book pp. 423 to 425)

30.4.1 Calculate the area of the following triangles.

(a) Sides of 20 cm and 14 cm that enclose an angle of 30°.

(b) Sides of 12 mm and 12 mm that enclose an angle of 80°.

(c) Sides of 35 cm and 40 cm that enclose an angle of 10°.

(d) Sides of 8 cm and 6 cm that enclose an angle of 135°.

30.4.2 Calculate the value of x in each of the following.

(a) A triangle with sides of 12 cm and 16 cm that enclose an angle of x°. Area = 40 cm².

(b) A triangle with a side of 9 cm and a side of x cm that enclose an angle of 160°. Area = 20 cm².

(c) A triangle with sides of 15 cm and x cm that enclose an angle of 60°. Area = 150 cm².

(d) A triangle with sides of 8 cm and 14 cm that enclose an angle of x°. Area = 50 cm².

30.4.3 A straight stretch of coast AB is shown in the diagram. A ship S at sea is 1.1 km from A. If angle SAB = 25°, calculate the shortest distance between the ship and the coast, giving your answer to the nearest metre.

30.4.4 The four corners, A, B, C and D, of a large nature reserve form a quadrilateral and are shown in the diagram. AC = 70 km, AD = 45 km and AB = 60 km. If angle BAC = 20° and angle BAD = 50°, calculate the area of the nature reserve, giving your answer to the nearest square kilometre. (In the diagram the line AC lies between AB and AD, with AB and AD on opposite sides of it.)

30.4.5 ABCD is a school playing field. The following lengths are known: OA = 83 m, OB = 122 m, OC = 106 m, OD = 78 m. Calculate the area of the school playing field to the nearest 100 m². (In the diagram O is a point inside the field joined to the corners. The angles at O are: AOB = 100°, BOC = 60°, COD = 130° and DOA = 70°.)

30.4.6 The roof of a garage has a slanting length of 3 m and makes an angle of 120° at its vertex. The height of the walls of the garage is 4 m and its depth is 9 m. (The roof is a triangular prism: its end is an isosceles triangle with two slanting sides of 3 m meeting at 120°.) Calculate:

(a) the cross-sectional area of the roof,

(b) the volume occupied by the whole garage.

Exercise 30.5 · Trigonometry in three dimensions (book pp. 427 to 429)

30.5.1 (In each cuboid and cube the bottom face is EFGH and the top face is ABCD, with A above E, B above F, C above G and D above H.) A cube of edge 4 cm (HG = 4 cm, GF = 4 cm, height 4 cm).

(a) Calculate the length HF.

(b) Calculate the length HB.

(c) Calculate the angle BHG.

30.5.2 A cuboid with HG = 5 cm, GF = 3 cm and height 2 cm.

(a) Calculate the length CA.

(b) Calculate the length CE.

(c) Calculate the angle ACE.

30.5.3 In the cuboid with HG = 4 cm, GF = 5 cm and height 12 cm:

(a) Calculate the length EG.

(b) Calculate the length AG.

(c) Calculate the angle AGE.

30.5.4 In the cuboid with HG = 3 cm, GF = 2 cm and height 7 cm, calculate:

(a) the angle BCE,

(b) the angle GFH.

30.5.5 The diagram shows a right pyramid where A is vertically above X. The base is DEBC (a rectangle) with X at its centre, DE = 6 cm, EB = 4 cm, and the sloping edge AB = 10 cm.

(i) Calculate the length DB.

(ii) Calculate the angle DAX.

(i) Calculate the angle CED.

(ii) Calculate the angle DBA.

30.5.6 The diagram shows a right pyramid where A is vertically above X. The base is DEBC (a rectangle) with X at its centre, DE = 8 cm, EB = 3 cm, and the sloping edge AB = 9 cm.

(i) Calculate the length CE.

(ii) Calculate the angle CAX.

(i) Calculate the angle BDE.

(ii) Calculate the angle ADB.

30.5.7 In this cone the angle YXZ = 60°. X is the apex, Y is the centre of the base and Z is a point on the edge of the base. The sloping side XZ = 13 cm. Calculate:

(a) the length XY,

(b) the length YZ,

(c) the circumference of the base.

30.5.8 In this cone the angle XZY = 40°. X is the apex, Y is the centre of the base, and YZ = 9 cm is the radius of the base. Calculate:

(a) the length XZ,

(b) the length XY.

30.5.9 One corner of a cuboid has been sliced off along the plane QTU. WU = 4 cm. The cuboid has edges of 10 cm, 8 cm and 6 cm (VW = 8 cm, the edge at the right-hand end is 6 cm, and the vertical edge is 10 cm). Calculate:

(a) the length of the three sides of the triangle QTU,

(b) the three angles P, Q and T in triangle PQT,

(c) the area of triangle PQT.

Exercise 30.6 · The angle between a line and a plane (book pp. 431 to 432)

30.6.1 A cuboid PQRSTUVW has the faces PQRS and TUVW opposite each other, with PT, QU, RV and SW as the joining edges. Name the projection of each line onto the given plane:

(a) TR onto RSWV

(b) TR onto PQUT

(c) SU onto PQRS

(d) SU onto TUVW

(e) PV onto QRVU

(f) PV onto RSWV

30.6.2 A cuboid HIJKLMNO is drawn as in the book diagram. Name the projection of each line onto the given plane:

(a) KM onto IJNM

(b) KM onto JKON

(c) KM onto HIML

(d) IO onto HLOK

(e) IO onto JKON

(f) IO onto LMNO

30.6.3 The cuboid PQRSTUVW is the same one as in question 1 (faces PQRS and TUVW opposite each other). Name the angle between the given line and plane:

(a) PT and PQRS

(b) PU and PQRS

(c) SV and PSWT

(d) RT and TUVW

(e) SU and QRVU

(f) PV and PSWT

30.6.4 In the cuboid with HG = 4 cm, GF = 3 cm and height 3 cm (bottom face EFGH, top face ABCD):

(a) Calculate the length BH.

(b) Calculate the angle between the line BH and the plane EFGH.

30.6.5 In the cuboid with HG = 8 cm, GF = 4 cm and height 5 cm (bottom face EFGH, top face ABCD):

(a) Calculate the length AG.

(b) Calculate the angle between the line AG and the plane EFGH.

(c) Calculate the angle between the line AG and the plane ADHE.

30.6.6 The diagram shows a right pyramid where A is vertically above X. The base is DEBC (a rectangle) with X at its centre, DE = 6 cm, EB = 3 cm and the sloping edge AB = 7 cm.

(a) Calculate the length BD.

(b) Calculate the angle between AB and CBED.

30.6.7 The diagram shows a right pyramid where U is vertically above X. The base is WVYZ (a rectangle) with X at its centre, ZY = 6 cm, YV = 5 cm and the sloping edge UV = 12 cm.

(a) Calculate the length WY.

(b) Calculate the length UX.

(c) Calculate the angle between UX and UZY.

30.6.8 ABCD and EFGH are square faces lying parallel to each other. AB = 10 cm, BC = 10 cm, HG = 6 cm, GF = 6 cm and the sloping edge CG = 8 cm. Calculate:

(a) the length DB,

(b) the length HF,

(c) the vertical height of the object,

(d) the angle DH makes with the plane ABCD.

30.6.9 ABCD and EFGH are square faces lying parallel to each other. AB = 12 cm, BC = 12 cm, HG = 4 cm, GF = 4 cm and the sloping edge CG = 9 cm. Calculate:

(a) the length AC,

(b) the length EG,

(c) the vertical height of the object,

(d) the angle CG makes with the plane EFGH.

Student assessment 1 (book pp. 433 to 434)

SA1.1 The triangle PQR has PR = 7 cm, PQ = 8 cm and angle PQR = 40°.

(a) Calculate the two possible values for angle PRQ.

(b) Calculate the shortest distance possible of R from side PQ.

SA1.2 Calculate the size of the obtuse angle marked θ° in the triangle. (The triangle has an angle of 30°, the side opposite the 30° angle is 18 cm, and the side opposite the angle θ° is 26 cm.)

SA1.3 For the cuboid with HG = 10 cm, GF = 6 cm and height 4 cm (bottom face EFGH, top face ABCD), calculate:

(a) the length EG,

(b) the length EC,

(c) angle BEC.

SA1.4 For the quadrilateral JKLM, JK = 25 m, KL = 12 m, LM = 20 m, angle KJM = 70° and angle JKL = 42°. Calculate:

(a) the length JL,

(b) angle KJL,

(c) the length JM,

(d) the area of JKLM.

SA1.5 For the square-based right pyramid ABCDE with A the apex, the base has sides of 9 cm and the sloping edges are 16 cm. Calculate:

(a) the length BD,

(b) angle ABD,

(c) the area of ΔABD,

(d) the vertical height of the pyramid.

Student assessment 2 (book pp. 434 to 435)

SA2.1 Two points X and Y are 1000 m apart on a straight piece of coastline. A buoy Z is out at sea. XZ = 850 m and YZ = 625 m. A girl wishes to swim from the coast to the buoy.

(a) What is the shortest distance she will have to swim? Give your answer to 3 s.f.

(b) How far along the coast from Y will she have to set off in order to swim the least distance?

SA2.2 Using the triangular prism (the bottom face CDEF is a rectangle with ED = 9 cm and DC = 5 cm; AE = 6 cm is vertical with a right angle at E; ABFE is a vertical face), calculate:

(a) the length AD,

(b) the length AC,

(c) the angle AC makes with the plane CDEF,

(d) the angle AC makes with the plane ABFE.

SA2.3 For the triangle PQR, with S a point on PR, PQ = 17 cm, angle QSR = 120°, the marks on the figure show PS = SQ, and angle PQR is a right angle, calculate:

(a) the length PS,

(b) angle QRS,

(c) the length SR.

SA2.4 The cuboid (bottom face EFGH, top face ABCD, HG = 5 cm, GF = 6 cm and height 8 cm) has one of its corners removed to leave a flat triangle BDC. Calculate:

(a) length DC,

(b) length BC,

(c) length DB,

(d) angle CBD,

(e) the area of ΔBDC,

(f) the angle AC makes with the plane AEHD.

Check yourself

C1 In triangle ABC, angle A = 48°, angle B = 65° and a = 12 cm. Calculate b. [3]

C2 Sides AB = 8 cm and AC = 11 cm enclose an angle of 70°. Calculate BC and the area of the triangle. [4]

C3 A triangle has sides 6 cm, 9 cm and 10 cm. Calculate its largest angle. [3]

C4 A cuboid is 6 cm long, 8 cm wide and 5 cm high. Calculate the length of the longest diagonal and the angle it makes with the base. [4]

C5 A triangle has sides 9 cm and 12 cm and an area of 40 cm², and the angle between those sides is acute. Calculate that angle. [3]

Chapter quiz

Q1 Which is the sine rule for triangle ABC (a is opposite A, b is opposite B)?

A	B	C	D
a/sin A = b/sin B	a sin A = b sin B	sin A/b = sin B/a	a/cos A = b/cos B

Q2 In triangle ABC, A = 30°, B = 45° and a = 8 cm. What is b to 1 d.p.?

A	B	C	D
5.7 cm	6.0 cm	9.8 cm	11.3 cm

Q3 You know two sides and the angle between them. Which rule finds the third side?

A	B	C	D
The sine rule	The cosine rule	Only Pythagoras	Only tan

Q4 Two sides of 5 cm and 8 cm enclose an angle of 60°. How long is the third side?

A	B	C	D
6 cm	7 cm	9 cm	13 cm

Q5 A triangle has sides 3 cm, 4 cm and 5 cm. What does the cosine rule give for the largest angle?

A	B	C	D
30°	53.1°	90°	120°

Q6 The sine rule gives sin B = 0.8 with B possibly obtuse. What are the two possible values of B?

A	B	C	D
53.1° and 126.9°	53.1° and 36.9°	36.9° and 143.1°	53.1° and 306.9°

Q7 Two sides of 10 cm and 12 cm enclose an angle of 30°. What is the area of the triangle?

A	B	C	D
30 cm²	60 cm²	52 cm²	120 cm²

Q8 What is the shortest distance from a point P to a straight line?

A	B	C	D
Along a slanting line	To the nearest marked point	Along the perpendicular from P	Twice the height

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Q9 A cuboid is 3 cm by 4 cm by 12 cm. How long is its longest diagonal?

A	B	C	D
13 cm	19 cm	5 cm	12.6 cm

Q10 In a 3D problem, what is the angle between a line and a plane?

A	B	C	D
The angle between the line and the plane's edge	The angle between the line and its projection on the plane	The angle between the line and the vertical	The angle at the corner of the plane

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Q11 A cube has edges of 4 cm. What angle does its longest diagonal make with the base?

A	B	C	D
35.3°	45°	54.7°	60°

Q12 Angles of 40° and 60° are known, with a side of 10 cm opposite the 40° angle. What is the third angle?

A	B	C	D
70°	80°	90°	100°

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-30-further-trigonometry/