C29

Trigonometry

CHAPTER ASSIGNMENT · 371 MARKS

Name _____ Class _____ No. _____ Date _____

Give your answers exactly or correct to three significant figures as appropriate. Answers in degrees should be given to one decimal place. Diagrams are not drawn to scale. Sketch each triangle and label opposite, adjacent and hypotenuse before you start.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 29 (Trigonometry), for classroom use. The book's diagrams cannot be printed here, so each one is described in brackets (the PDF page shows the drawing); the descriptions are Guru Jazzy's own. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 29.1 · The tangent ratio

29.1.1 Calculate the length of the side marked x cm in each of the diagrams below.

(a) (Triangle ABC, right angle at B. Angle A = 20°. AB = 5 cm. BC = x cm.) [2]

(b) (Triangle ABC, right angle at C. Angle A = 30°. AC = 7 cm. BC = x cm.) [2]

(c) (Triangle PQR, right angle at Q. Angle P = 58°. PQ = 12 cm. QR = x cm.) [2]

(d) (Triangle LMN, right angle at M. Angle N = 18°. MN = 15 cm. LM = x cm.) [2]

(e) (Triangle ABC, right angle at B. Angle C = 75°. BC = 10 cm. AB = x cm.) [2]

(f) (Triangle PQR, right angle at Q. Angle R = 60°. QR = 8 cm. QP = x cm.) [2]

29.1.2 Calculate the length of the side marked x cm in each of the diagrams below.

(a) (Triangle ABC, right angle at C. Angle B = 40°. AC = 12 cm. BC = x cm.) [2]

(b) (Triangle PQR, right angle at Q. Angle P = 38°. QR = 7 cm. PQ = x cm.) [2]

(c) (Triangle DEF, right angle at E. Angle D = 65°. EF = 20 cm. DE = x cm.) [2]

(d) (Triangle LMN, right angle at M. Angle N = 26°. LM = 2 cm. MN = x cm.) [2]

(e) (Triangle LMN, right angle at L. Angle N = 25°. LM = 6.5 cm. LN = x cm.) [2]

(f) (Triangle ABC, right angle at C. Angle A = 56°. BC = 9.2 cm. AC = x cm.) [2]

29.1.3 Calculate the size of the marked angle x° in each of the following diagrams.

(a) (Triangle PQR, right angle at Q. QP = 7 cm, QR = 6 cm. Angle x is at R.) [2]

(b) (Triangle DEF, right angle at E. DE = 10.5 cm, EF = 13 cm. Angle x is at D.) [2]

(c) (Triangle ABC, right angle at B. BC = 12 cm, BA = 15 cm. Angle x is at C.) [2]

(d) (Triangle PQR, right angle at R. PR = 8 cm, QR = 4 cm. Angle x is at Q.) [2]

(e) (Triangle ABC, right angle at B. BC = 7.5 cm, BA = 6.2 cm. Angle x is at A.) [2]

(f) (Triangle LMN, right angle at N. LN = 1 cm, MN = 3 cm. Angle x is at L.) [2]

Exercise 29.2 · The sine ratio

29.2.1 Calculate the length of the marked side in each of the following diagrams.

(a) (Triangle LMN, right angle at M. Angle L = 24°. The hypotenuse LN = 6 cm. MN = l cm.) [2]

(b) (Triangle PQR, right angle at Q. Angle R = 60°. QP = 16 cm. The hypotenuse PR = q cm.) [2]

(c) (Triangle ABC, right angle at B. Angle C = 49°. The hypotenuse AC = 8.2 cm. AB = c cm.) [2]

(d) (Triangle XYZ, right angle at Y. Angle X = 55°. YZ = 2 cm. The hypotenuse XZ = y cm.) [2]

(e) (Triangle JKL, right angle at K. Angle J = 22°. LK = 16.4 cm. The hypotenuse JL = k cm.) [2]

(f) (Triangle ABC, right angle at B. Angle C = 45°. The hypotenuse AC = 45 cm. AB = c cm.) [2]

29.2.2 Calculate the size of the angle marked x in each of the following diagrams.

(a) (Triangle ABC, right angle at B. BA = 5 cm, BC = 8 cm. Angle x is at C.) [2]

(b) (Triangle DEF, right angle at E. The hypotenuse DF = 16 cm, EF = 12 cm. Angle x is at D.) [2]

(c) (Triangle EFG, right angle at F. The hypotenuse EG = 6.8 cm, GF = 4.2 cm. Angle x is at E.) [2]

(d) (Triangle LMN, right angle at L. LM = 7.1 cm. The hypotenuse MN = 9.3 cm. Angle x is at N.) [2]

(e) (Triangle PQR, right angle at Q. The hypotenuse PR = 26 cm, QR = 14 cm. Angle x is at P.) [2]

(f) (Triangle ABC, right angle at B. AB = 0.3 m. The hypotenuse AC = 1.2 m. Angle x is at C.) [2]

Exercise 29.3 · The cosine ratio

29.3.1 Calculate the marked side or angle in each of the following diagrams.

(a) (Triangle ABC, right angle at B. Angle C = 26°. The hypotenuse AC = 40 cm. BC = a cm.) [2]

(b) (Triangle XYZ, right angle at Y. Angle X = 15°. XY = 14.6 cm. The hypotenuse XZ = y cm.) [2]

(c) (Triangle EFG, right angle at F. The hypotenuse EG = 18 cm, EF = 12 cm. Angle x is at E.) [2]

(d) (Triangle LMN, right angle at L. LM = 8.1 cm. The hypotenuse MN = 52.3 cm. Angle a is at M.) [2]

(e) (Triangle XYZ, right angle at X. Angle Y = 56°. The hypotenuse YZ = 12 cm. XY = z cm.) [2]

(f) (Triangle HIJ, right angle at I. Angle J = 27°. JI = 15 cm. The hypotenuse JH = i cm.) [2]

(g) (Triangle XYZ, right angle at Y. XY = 0.2 m. The hypotenuse XZ = 0.6 m. Angle e is at X.) [2]

(h) (Triangle ABC, right angle at C. Angle B = 81°. The hypotenuse BA = 13.7 cm. BC = a cm.) [2]

Exercise 29.4 · Pythagoras' theorem

29.4.1 In each of the diagrams, use Pythagoras' theorem to calculate the length of the marked side.

(a) (A right-angled triangle with the two shorter sides 4 cm and 3 cm. The hypotenuse is a cm.) [2]

(b) (A right-angled triangle with the two shorter sides 9 mm and 7 mm. The hypotenuse is b cm.) [2]

(c) (A right-angled triangle with the hypotenuse 15 cm and one shorter side 9 cm. The other shorter side is c cm.) [2]

(d) (A right-angled triangle with the hypotenuse 20 cm and one shorter side 15 cm. The other shorter side is d cm.) [2]

29.4.2 In each of the diagrams, use Pythagoras' theorem to calculate the length of the marked side.

(a) (A quadrilateral made of two right-angled triangles sharing a diagonal. The first has legs 9 cm and 5 cm, so its hypotenuse is the shared diagonal. The second has the shared diagonal as one leg and 4 cm as the other leg, with the right angle at the end of the 9 cm side. Its hypotenuse is e cm.) [3]

(b) (Two right-angled triangles sharing a diagonal. The first has legs 12 cm and 5 cm. The second has the shared diagonal as one leg and 7 cm as the other, with its right angle between them. Its hypotenuse is f cm.) [3]

(c) (A large right-angled triangle with height 8 cm and hypotenuse 16 cm. A line from the top vertex to the base has length 10 cm. It cuts the base so that the part of the base on the left of the cut is g cm.) [3]

(d) (A triangle with a perpendicular from its top vertex to the base. The foot of the perpendicular splits the base into 3 cm on the left and 6 cm on the right. The right-hand slanting side is 9 cm. The left-hand slanting side is h cm.) [3]

(e) (A fan of three right-angled triangles meeting at one point. The first has two legs of 1 cm. The second has the first one's hypotenuse as a leg and an outer side of $\sqrt{2}$ cm as the other leg. The third has the second one's hypotenuse as a leg and an outer side of $\sqrt{5}$ cm as the other leg. The final hypotenuse is j cm.) [4]

(f) (Two overlapping right-angled triangles with right angles at the bottom corners of a shared base. The left one has height 4 cm and a hypotenuse of 8 cm. The right one has height 12 cm. The line from the bottom-left corner to the top of the right triangle is k cm.) [4]

29.4.3 Villages A, B and C lie on the edge of the Namib Desert. Village A is 30 km due north of village C. Village B is 65 km due east of A. Calculate the shortest distance between villages C and B, giving your answer to the nearest 0.1 km. (A is at the top left, B at the top right and C directly below A.) [3]

29.4.4 Town X is 54 km due west of town Y. The shortest distance between town Y and town Z is 86 km. If town Z is due south of X, calculate the distance between X and Z, giving your answer to the nearest kilometre. [3]

29.4.5 Village B is on a bearing of 135° and at a distance of 40 km from village A, as shown. Village C is on a bearing of 225° and a distance of 62 km from village A. (A diagram shows A with a north arrow, B to the south-east of A and C to the south-west of A.)

(a) Show that triangle ABC is right-angled. [2]

(b) Calculate the distance from B to C, giving your answer to the nearest 0.1 km. [3]

29.4.6 Two boats set off from X at the same time (a diagram shows X with a north arrow, A to the north-west and B to the south-west). Boat A sets off on a bearing of 325° and with a velocity of 14 km/h. Boat B sets off on a bearing of 235° with a velocity of 18 km/h. Calculate the distance between the boats after they have been travelling for 2.5 hours. Give your answer to the nearest metre. [5]

29.4.7 A boat sets off on a trip from S. It heads towards B, a point 6 km away and due north. At B it changes direction and heads towards point C, 6 km away from and due east of B. At C it changes direction once again and heads on a bearing of 135° towards D, which is 13 km from C.

(a) Calculate the distance between S and C to the nearest 0.1 km. [3]

(b) Calculate the distance the boat will have to travel if it is to return to S from D. [4]

29.4.8 Two trees are standing on flat ground. (The taller tree is on the left, the smaller on the right. Two crossing lines show the 15 m and 20 m distances.) The height of the smaller tree is 7 m. The distance between the top of the smaller tree and the base of the taller tree is 15 m. The distance between the top of the taller tree and the base of the smaller tree is 20 m.

(a) Calculate the horizontal distance between the two trees. [3]

(b) Calculate the height of the taller tree. [3]

Exercise 29.5 · Mixed problems

29.5.1 By using Pythagoras' theorem, trigonometry or both, calculate the marked value in each of the following diagrams. In each case give your answer to 1 d.p.

(a) (Triangle ABC, right angle at B. AB = 6 cm. The hypotenuse AC = 8.7 cm. Angle x is at C.) [2]

(b) (Triangle JKL, right angle at K. Angle J = 38°. KL = 15.2 cm. JK = l cm.) [2]

(c) (Triangle LMN, right angle at N. LM = 17.4 cm, LN = 4.8 cm. MN = l cm.) [2]

(d) (Triangle XYZ, right angle at X. XY = 14 cm. YZ = 19 cm. Angle z is at Y.) [2]

29.5.2 A sailing boat sets off from a point X and heads towards Y, a point 17 km north. At point Y, it changes direction and heads towards point Z, a point 12 km away on a bearing of 090°. Once at Z, the crew want to sail back to X. Calculate:

(a) the distance ZX, [3]

(b) the bearing of X from Z. [3]

29.5.3 An aeroplane sets off from G on a bearing of 024° towards H, a point 250 km away. At H, it changes course and heads towards J, which is 180 km away on a bearing of 055°. (A diagram shows G at the bottom with a north arrow, H above and to the right of G, and J above and to the right of H.)

(a) How far is H to the north of G? [2]

(b) How far is H to the east of G? [2]

(c) How far is J to the north of H? [2]

(d) How far is J to the east of H? [2]

(e) What is the shortest distance between G and J? [3]

(f) What is the bearing of G from J? [3]

29.5.4 Two trees are standing on flat ground. The angle of elevation of their tops from a point X on the ground is 40°. If the horizontal distance between X and the small tree is 8 m and the distance between the tops of the two trees is 20 m, calculate: (The two tops and X lie on one straight line. The line from X rises at 40° to the horizontal, passes the top of the small tree 8 m from X horizontally, and reaches the top of the tall tree. The 20 m is the length of line between the two tops.)

(a) the height of the small tree, [2]

(b) the height of the tall tree, [3]

(c) the horizontal distance between the trees. [3]

29.5.5 PQRS is a quadrilateral. The sides RS and QR are the same length. The sides QP and RS are parallel. (S is at the left, Q at the right, P above the line SQ and R below it. SQ = 12 cm and QR = 9 cm.) Calculate:

(a) angle SQR, [2]

(b) angle PSQ, [3]

(c) length PQ, [3]

(d) length PS, [3]

(e) the area of PQRS. [3]

Exercise 29.6 · Angles of elevation and depression

29.6.1 A and B are two villages. If the horizontal distance between them is 12 km and the vertical distance between them is 2 km, calculate: (B is on higher ground than A.)

(a) the shortest distance between the two villages, [2]

(b) the angle of elevation of B from A. [2]

29.6.2 X and Y are two towns. If the horizontal distance between them is 10 km and the angle of depression of Y from X is 7°, calculate: (X is on higher ground than Y.)

(a) the shortest distance between the two towns, [2]

(b) the vertical height between the two towns. [2]

29.6.3 A girl standing on a hill at A can see a small boat at a point B on a lake. (The girl is on a hill, 50 m above the lake, and the boat is 120 m away horizontally.) If the girl is at a height of 50 m above B and at a horizontal distance of 120 m away from B, calculate:

(a) the angle of depression of the boat from the girl, [2]

(b) the shortest distance between the girl and the boat. [2]

29.6.4 Two hot air balloons are 1 km apart in the air. If the angle of elevation of the higher from the lower balloon is 20°, calculate, giving your answers to the nearest metre: (The 1 km is the straight-line distance between the balloons.)

(a) the vertical height between the two balloons, [2]

(b) the horizontal distance between the two balloons. [2]

29.6.5 A boy X can be seen by two of his friends, Y and Z, who are swimming in the sea. (X is on a cliff, 40 m above the sea. Y and Z are in the sea in line with X's foot, with Z nearer the cliff than Y.) If the angle of elevation of X from Y is 23° and from Z is 32°, and the height of X above Y and Z is 40 m, calculate: Note: XYZ is a vertical plane.

(a) the horizontal distance between X and Z, [2]

(b) the horizontal distance between Y and Z. [3]

29.6.6 A plane is flying at an altitude of 6 km directly over the line AB. It spots two boats, A and B, on the sea. If the angles of depression of A and B from the plane are 60° and 30° respectively, calculate the horizontal distance between A and B. [4]

29.6.7 Two planes are flying directly above each other. A person standing at P can see both of them. The horizontal distance between the two planes and the person is 2 km. If the angles of elevation of the planes from the person are 65° and 75°, calculate:

(a) the altitude at which the higher plane is flying, [2]

(b) the vertical distance between the two planes. [3]

29.6.8 Three villages, A, B and C, can see each other across a valley. The horizontal distance between A and B is 8 km, and the horizontal distance between B and C is 12 km. The angle of depression of B from A is 20° and the angle of elevation of C from B is 30°. (A is on the left slope, B is at the bottom of the valley and C is on the right slope.) Calculate, giving all answers to 1 d.p.: Note: A, B and C are in the same vertical plane.

(a) the vertical height between A and B, [2]

(b) the vertical height between B and C, [2]

(c) the angle of elevation of C from A, [3]

(d) the shortest distance between A and C. [3]

29.6.9 Using binoculars, three people, P, Q and R, can see each other across a valley. The horizontal distance between P and Q is 6.8 km and the horizontal distance between Q and R is 10 km. If the shortest distance between P and Q is 7 km and the angle of depression of Q from R is 15°, calculate, giving appropriate answers: (P is on the left slope, Q is at the bottom of the valley and R is on the right slope.) Note: P, Q and R are in the same vertical plane.

(a) the vertical height between Q and R, [2]

(b) the vertical height between P and R, [3]

(c) the angle of elevation of R from P, [3]

(d) the shortest distance between P and R. [3]

29.6.10 Two people, A and B, are standing either side of a transmission mast. A is 130 m away from the mast and B is 200 m away. If the angle of elevation of the top of the mast from A is 60°, calculate: (A and B are on level ground with the mast between them.)

(a) the height of the mast to the nearest metre, [2]

(b) the angle of elevation of the top of the mast from B. [2]

Exercise 29.7 · The sine curve

29.7.1 Express each of the following in terms of the sine of another angle between 0° and 180°:

(a) sin 60°	(b) sin 80°	(c) sin 115°	(d) sin 140°	(e) sin 128°	(f) sin 167°

29.7.2 Express each of the following in terms of the sine of another angle between 0° and 180°:

(a) sin 35°	(b) sin 50°	(c) sin 30°	(d) sin 48°	(e) sin 104°	(f) sin 127°

29.7.3 Find the two angles between 0° and 180° which have the following sine. Give each angle to the nearest degree.

(a) 0.33	(b) 0.99	(c) 0.09	(d) 0.95	(e) 0.22	(f) 0.47

29.7.4 Find the two angles between 0° and 180° which have the following sine. Give each angle to the nearest degree.

(a) 0.94	(b) 0.16	(c) 0.80	(d) 0.56	(e) 0.28	(f) 0.33

Exercise 29.8 · The cosine and tangent curves

29.8.1 Express each of the following in terms of the cosine of another angle between 0° and 180°:

(a) cos 20°	(b) cos 85°	(c) cos 32°	(d) cos 95°	(e) cos 147°	(f) cos 106°

29.8.2 Express each of the following in terms of the cosine of another angle between 0° and 180°:

(a) cos 98°	(b) cos 144°	(c) cos 160°	(d) cos 143°	(e) cos 171°	(f) cos 123°

29.8.3 Express each of the following in terms of the cosine of another angle between 0° and 180°:

(a) −cos 100°	(b) cos 90°	(c) −cos 110°	(d) −cos 45°	(e) −cos 122°	(f) −cos 25°

29.8.4 The cosine of which acute angle has the same value as each of the following?

(a) cos 125°	(b) cos 107°	(c) −cos 120°	(d) −cos 98°	(e) −cos 92°	(f) −cos 110°

29.8.5 Explain with reference to a right-angled triangle why the tangent of 90° is undefined. [2]

Exercise 29.9 · Solving trigonometric equations

29.9.1 Solve each of the following equations, giving all the solutions in the range $0 \leq \theta \leq 360$.

(a) $\sin\theta = 1/4$	(b) $\cos\theta = 1/\sqrt{2}$	(c) $\sin\theta = -1/2$
(d) $\tan\theta = -\sqrt{3}$	(e) $5\cos\theta + 1 = 2$	(f) $1/2\tan\theta + 2 = 1$

29.9.2 In the triangle (a right-angled triangle with the two shorter sides 3 and 4, and θ the angle opposite the side of length 3), tan θ = 3/4. Deduce, without a calculator, the value of:

(a) sin θ [2]

(b) cos θ [2]

29.9.3 In the triangle below (a right-angled triangle with the hypotenuse 10 and a shorter side 3, and θ the angle opposite the side of length 3), sin θ = 3/10. Deduce, without a calculator, the exact value of:

(a) cos θ [2]

(b) tan θ [2]

29.9.4 By using the triangle (a right-angled triangle with the sides marked Opposite, Adjacent and Hypotenuse and the angle θ at the end of the adjacent side) as an aid, explain why the solution to the equation sin θ = cos θ occurs when θ = 45°. [3]

Student assessment 1

SA1.1 Calculate the length of the side marked x cm in each of the following.

(a) (A right-angled triangle. Angle 30°. The hypotenuse is 8 cm. x is the side opposite the 30° angle.) [2]

(b) (A triangle with a right angle at the top. The side 15 cm is opposite the 20° angle. x is the hypotenuse.) [2]

(c) (A triangle with a right angle at the bottom. The angle 60° is at the left. The side 10.4 cm is next to the 60° angle. x is the hypotenuse.) [2]

(d) (A triangle with a right angle at the top. The angle 50° is at the right. The side 3 cm is opposite the 50° angle. x is the hypotenuse.) [2]

SA1.2 Calculate the size of the angle marked θ° in each of the following.

(a) (A right-angled triangle. The hypotenuse is 15 cm, the side opposite θ is 9 cm.) [2]

(b) (A right-angled triangle. The side next to the right angle on the left is 4.2 cm, the side along the bottom is 6.3 cm, and θ is at the top.) [2]

(c) (A right-angled triangle. The right angle is at the top left. The top side is 3 cm, the left side is 5 cm, and θ is at the bottom.) [2]

(d) (A right-angled triangle. The hypotenuse is 14.8 cm, the side next to θ is 12.3 cm.) [2]

SA1.3 Calculate the length of the side marked q cm in each of the following.

(a) (A right-angled triangle with the two shorter sides 3 cm and 4 cm. q is the hypotenuse.) [2]

(b) (A right-angled triangle with the hypotenuse 12 cm and one shorter side 10 cm. q is the other shorter side.) [2]

(c) (A right-angled triangle with the top side 3 cm and a 65° angle at the end of that side. A line from the bottom vertex to the end of the 3 cm side is the hypotenuse of that triangle. A second right-angled triangle, with the right angle at the end of the 3 cm side, has a shorter side of 6 cm and its hypotenuse is q cm.) [4]

(d) (Two overlapping right-angled triangles sharing a base. The left one has height 18 cm and the angle 25° at the bottom right between the base and its hypotenuse. The right one has a hypotenuse of 48 cm. q is the height of the right one.) [4]

SA1.4 In the triangle below (the hypotenuse is 6 and the side next to θ is 5), cos θ = 5/6. Showing your working clearly, deduce the exact value of:

(a) sin θ [2]

(b) tan θ [2]

Student assessment 2

SA2.1 A map shows three towns A, B and C. Town A is due north of C. Town B is due east of A. The distance AC is 75 km and the bearing of C from B is 245°. Calculate, giving your answers to the nearest 100 m: (A is at the top left, B at the top right and C directly below A.)

(a) the distance AB, [3]

(b) the distance BC. [3]

SA2.2 Two trees stand 16 m apart. Their tops make an angle of θ° at point A on the ground. (The shorter tree is 5 m tall and the taller is 7.5 m tall. The shorter tree is x m from A and the taller one is 16 m further along. The tops and A are in one straight line.)

(a) Express θ° in terms of the height of the shorter tree and its distance x metres from point A. [1]

(b) Express θ° in terms of the height of the taller tree and its distance from A. [1]

(c) Form an equation in terms of x. [1]

(d) Calculate the value of x. [3]

(e) Calculate the value θ. [2]

SA2.3 Two boats X and Y, sailing in a race, are shown in the diagram. Boat X is 145 m due north of a buoy B. Boat Y is due east of buoy B. Boats X and Y are 320 m apart. Calculate:

(a) the distance BY, [3]

(b) the bearing of Y from X, [3]

(c) the bearing of X from Y. [2]

SA2.4 Two hawks P and Q are flying vertically above one another. Hawk Q is 250 m above hawk P. They both spot a snake at R. (R is on the ground 2.8 km horizontally from the hawks. The angle of 35° is marked at R, between the ground and the line RP.) Using the information given, calculate:

(a) the height of P above the ground, [2]

(b) the distance between P and R, [2]

(c) the distance between Q and R. [3]

SA2.5 Solve the following trigonometric equations, giving all the solutions in the range $0 \leq x \leq 360$.

(a) $\sin x = 2/5$	(b) $\tan x = -\sqrt{3}/3$	(c) $\cos x = -0.1$	(d) $\sin x = 1$

Student assessment 3

SA3.1 Explain, with the aid of a graph, why the equation cos x = 3/2 has no solutions. [2]

SA3.2 The cosine of which other angle between 0 and 180° has the same value as:

(a) cos 128° [1]

(b) −cos 80°? [1]

SA3.3 A circle of radius 3 cm, centre at O, is shown on the axes below. (The circle is centred on the origin of a coordinate grid.) The points A and B lie where the circumference of the circle intersects the x-axis. Point C is free to move on the circumference of the circle.

(a) Deduce, justifying your answer, the size of angle ACB. [2]

(b) If BC = 3 cm, calculate the possible coordinates of point C, giving your answers in exact form. [4]

SA3.4 The Great Pyramid at Giza is 146 m high. Two people A and B are looking at the top of the pyramid. The angle of elevation of the top of the pyramid from B is 12°. The distance between A and B is 25 m. (A and B stand in a line on the ground with B nearer the pyramid. The angle of elevation of the top from A is θ.) If both A and B are 1.8 m tall, calculate: Note: A, B and the top of the pyramid are in the same vertical plane.

(a) the distance on the ground from B to the centre of the base of the pyramid, [3]

(b) the angle of elevation θ of the top of the pyramid from A, [3]

(c) the distance between A and the top of the pyramid. [3]

SA3.5 Two hot air balloons A and B are travelling in the same horizontal direction as shown in the diagram below. A is travelling at 2 m/s and B at 3 m/s. Their heights above the ground are 1.6 km and 1 km, respectively. (Both balloons are moving to the left. A is 4 km to the left of B. B is directly above a point X on the ground.) At midday, their horizontal distance apart is 4 km and balloon B is directly above a point X on the ground. Calculate:

(a) the angle of elevation of A from X at midday, [2]

(b) the angle of depression of B from A at midday, [2]

(c) their horizontal distance apart at 12 30, [3]

(d) the angle of elevation of B from X at 12 30, [3]

(e) the angle of elevation of A from B at 12 30, [3]

(f) how much closer A and B are at 12 30 compared with midday. [3]

SA3.6 (a) On one diagram, plot the graph of $y = \sin \theta°$ and the graph of $y = \cos \theta°$, for $0° \leq \theta° \leq 180°$. [4]

(b) Use your graph to find the angles for which $\sin \theta° = \cos \theta°$. [2]

Check yourself

1 In a right-angled triangle, the side opposite the 37° angle is 9 cm. Calculate the hypotenuse, to 3 s.f. [3]

2 A ladder 6 m long leans against a wall and makes an angle of 70° with the ground. How high up the wall does it reach? Give your answer to 3 s.f. [3]

3 A right-angled triangle has sides 7 cm and 24 cm next to the right angle. Find the hypotenuse, then the smaller acute angle. [4]

4 From a point 50 m from the foot of a tower, the angle of elevation of the top is 32°. Find the height of the tower to the nearest metre. [3]

5 Solve $\sin x = 0.6$ for $0° \le x \le 180°$, giving both angles to 1 d.p. [3]

6 Explain why you cannot use sin, cos or tan to find a side when you know two sides of a right-angled triangle and nothing else. Which rule do you use instead? [2]

Chapter quiz

Q1 Which ratio is sin θ in a right-angled triangle?

A	B	C	D
adjacent ÷ hypotenuse	opposite ÷ adjacent	opposite ÷ hypotenuse	hypotenuse ÷ opposite

Q2 A right-angled triangle has opposite 5 cm and adjacent 12 cm. What is the angle θ?

A	B	C	D
22.6°	67.4°	24.6°	0.4°

Q3 The hypotenuse is 10 cm and an angle is 30°. How long is the side opposite that angle?

A	B	C	D
5 cm	8.66 cm	20 cm	11.5 cm

Q4 The side next to a 60° angle is 8 cm. How long is the hypotenuse?

A	B	C	D
4 cm	16 cm	6.9 cm	13.9 cm

Q5 A right-angled triangle has hypotenuse 17 cm and one shorter side 8 cm. The other side is...

A	B	C	D
9 cm	15 cm	18.8 cm	12.1 cm

Q6 You are 40 m from a tree and its top is at an angle of elevation of 25°. How tall is the tree?

A	B	C	D
17.0 m	18.7 m	36.3 m	85.8 m

Q7 sin 40° = 0.643. Which other angle between 0° and 180° also has a sine of 0.643?

A	B	C	D
50°	130°	140°	320°

Q8 Which of these is equal to cos 70°?

A	B	C	D
cos 110°	−cos 110°	−cos 70°	cos 20°

Q9 Solve tan θ = 1 for 0° ≤ θ ≤ 360°.

A	B	C	D
45° only	45° and 135°	45° and 225°	45° and 315°

Q10 Which equation has no solutions at all?

A	B	C	D
sin x = 1	cos x = −1	tan x = 100	cos x = 1.5

Q11 From a cliff top, a boat is seen at an angle of depression of 20°. The cliff top from the boat is at...

A	B	C	D
an elevation of 20°	an elevation of 70°	a depression of 20°	an elevation of 160°

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-29-trigonometry/