C27

Perimeter, area and volume

CHAPTER ASSIGNMENT · 444 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Show your working. Diagrams are described in words and are not drawn to scale. You may use a calculator; give answers to 3 significant figures unless the question asks for another form.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 27 (Perimeter, area and volume), for classroom use. Questions that used a diagram in the book describe it in words (in brackets). The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 27.1 · The area of a triangle

27.1.1 Calculate the areas of the triangles below:

(a) (A right-angled triangle: the two sides next to the right angle are 3 cm and 4 cm.) [1]

(b) (A right-angled triangle: the two sides next to the right angle are 13 cm and 5 cm.) [1]

(c) (Base 8 cm, perpendicular height 5 cm.) [1]

(d) (Base 10 cm, perpendicular height 12 cm.) [1]

(e) (Base 24 cm, perpendicular height 9 cm.) [1]

(f) (Base 11 cm, perpendicular height 10 cm.) [1]

27.1.2 Calculate the areas of the shapes below:

(a) (A square of side 8 cm with a triangle on each of its four sides. Each triangle has a base of 8 cm and a perpendicular height of 4 cm.) [2]

(b) (A square of side 32 mm with a square of side 12 mm joined to its right-hand side.) [2]

(c) (A kite whose diagonals are 30 cm and 20 cm long.) [2]

(d) (A kite whose diagonals are 25 mm and 75 mm long.) [2]

Exercise 27.2 · Parallelograms and trapezia

27.2.0 Find the area of each of the following shapes:

(1) (A parallelogram with base 6.5 cm and perpendicular height 9 cm.) [2]

(2) (A trapezium with parallel sides 13 cm and 8 cm, 8 cm apart.) [2]

(3) (A shape with a flat base of 6.4 cm + 5.5 cm. On the right is a rectangle 5.5 cm wide and 11 cm tall. On the left is a trapezium 6.4 cm wide whose parallel vertical sides are 7.2 cm and 11 cm.) [3]

(4) (A parallelogram with top side 15 cm and perpendicular height 7 cm. Its bottom side, also 15 cm, is the base of a triangle below it with perpendicular height 7 cm.) [3]

Exercise 27.3 · Problems with area

27.3.1 Calculate a. (A trapezium with parallel sides a cm and 6 cm, 4 cm apart. Area = 20 cm^2.) [2]

27.3.2 If the areas of this trapezium and parallelogram are equal, calculate x. (The trapezium sits on top of the parallelogram. The trapezium has parallel sides 6 cm and 12 cm, 4 cm apart. The parallelogram has base 12 cm and perpendicular height x cm.) [3]

27.3.3 The end view of a house is as shown in the diagram (below). If the door has a width and height of 0.75 m and 2 m respectively, calculate the area of brickwork. (The end wall is a trapezium 5 m wide whose vertical sides are 6 m and 4 m tall. The door is a rectangle in the wall.) [3]

27.3.4 A garden in the shape of a trapezium is split into three parts: a flower bed in the shape of a triangle, a flower bed in the shape of a parallelogram and a section of grass in the shape of a trapezium, as shown (right). The area of the grass is two and a half times the total area of flower beds. Calculate:

(The garden is 8 m from front to back. Its back edge is 10 m. Its front edge is 4 m + x m + 3 m. The triangle flower bed is on the left with a base of 4 m, the parallelogram flower bed is on the right with a base of 3 m, and the grass trapezium is between them.)

(a) the area of each flower bed, [2]

(b) the area of grass, [1]

(c) the value of x. [3]

Exercise 27.4 · Circumference and area of a circle

27.4.1 Calculate the circumference of each circle, giving your answer to 3 s.f.

(a) radius 4 cm	(b) radius 3.5 cm	(c) diameter 9.2 cm	(d) radius 0.5 m

27.4.2 Calculate the area of each of the circles in Question 1. Give your answers to 3 s.f.

(a)	(b)	(c)	(d)

27.4.3 Calculate the radius of a circle when the circumference is:

(a) 15 cm	(b) π cm	(c) 4 m	(d) 8 mm

27.4.4 Calculate the diameter of a circle when the area is:

(a) 16 cm^2	(b) 9π cm^2	(c) 8.2 m^2	(d) 14.6 mm^2

Exercise 27.5 · Circle problems

27.5.1 The wheel of a car has an outer radius of 25 cm. Calculate:

(a) how far the car has travelled after one complete turn of the wheel, [2]

(b) how many times the wheel turns for a journey of 1 km. [2]

27.5.2 If the wheel of a bicycle has a diameter of 60 cm, calculate how far a cyclist will have travelled after the wheel has rotated 100 times. [3]

27.5.3 A circular ring has a cross-section as shown. If the outer radius is 22 mm and the inner radius 20 mm, calculate the cross-sectional area of the ring. Give your answer in terms of π. [3]

27.5.4 Four circles are drawn in a line and enclosed by a rectangle as shown. If the radius of each circle is 3 cm, calculate the unshaded area within the rectangle giving your answer in exact form. (Each circle touches its neighbours and the sides of the rectangle.) [4]

27.5.5 A garden is made up of a rectangular patch of grass and two semicircular vegetable patches. If the dimensions of the rectangular patch are 16 m (length) and 8 m (width) respectively, calculate in exact form: (The semicircles sit on the two 8 m ends.)

(a) the perimeter of the garden, [2]

(b) the total area of the garden. [2]

Exercise 27.6 · Surface area of a cuboid and a cylinder

27.6.1 Calculate the surface area of each of the following cuboids:

(a) l = 12 cm, w = 10 cm, h = 5 cm [2]

(b) l = 4 cm, w = 6 cm, h = 8 cm [2]

(c) l = 4.2 cm, w = 7.1 cm, h = 3.9 cm [2]

(d) l = 5.2 cm, w = 2.1 cm, h = 0.8 cm [2]

27.6.2 Calculate the height of each of the following cuboids:

(a) l = 5 cm, w = 6 cm, surface area = 104 cm^2 [2]

(b) l = 2 cm, w = 8 cm, surface area = 112 cm^2 [2]

(c) l = 3.5 cm, w = 4 cm, surface area = 118 cm^2 [2]

(d) l = 4.2 cm, w = 10 cm, surface area = 226 cm^2 [2]

27.6.3 Calculate the surface area of each of the following cylinders:

(a) r = 2 cm, h = 6 cm [2]

(b) r = 4 cm, h = 7 cm [2]

(c) r = 3.5 cm, h = 9.2 cm [2]

(d) r = 0.8 cm, h = 4.3 cm [2]

27.6.4 Calculate the height of each of the following cylinders. Give your answers to 1 d.p.

(a) r = 2.0 cm, surface area = 40 cm^2 [2]

(b) r = 3.5 cm, surface area = 88 cm^2 [2]

(c) r = 5.5 cm, surface area = 250 cm^2 [2]

(d) r = 3.0 cm, surface area = 189 cm^2 [2]

Exercise 27.7 · Surface area problems

27.7.1 Two cubes are placed next to each other. The length of each of the edges of the larger cube is 4 cm. If the ratio of their surface areas is 1 : 4, calculate:

(a) the surface area of the small cube, [2]

(b) the length of an edge of the small cube. [2]

27.7.2 A cube and a cylinder have the same surface area. If the cube has an edge length of 6 cm and the cylinder a radius of 2 cm, calculate:

(a) the surface area of the cube, [1]

(b) the height of the cylinder. [3]

27.7.3 Two cylinders have the same surface area. The shorter of the two has a radius of 3 cm and a height of 2 cm, and the taller cylinder has a radius of 1 cm. Calculate:

(a) the surface area of one of the cylinders in terms of π [2]

(b) the height of the taller cylinder. [3]

27.7.4 Two cuboids have the same surface area. The dimensions of one of them are: length = 3 cm, width = 4 cm and height = 2 cm. Calculate the height of the other cuboid if its length is 1 cm and its width is 4 cm. [4]

Exercise 27.8 · Volume and surface area of prisms

27.8.1 Calculate the volume of each of the following cuboids, where w, l and h represent the width, length and height respectively.

(a) w = 2 cm, l = 3 cm, h = 4 cm [1]

(b) w = 6 cm, l = 1 cm, h = 3 cm [1]

(c) w = 6 cm, l = 23 mm, h = 2 cm [2]

(d) w = 42 mm, l = 3 cm, h = 0.007 m [2]

27.8.2 Calculate the volume of each of the following cylinders, where r represents the radius of the circular face and h the height of the cylinder.

(a) r = 4 cm, h = 9 cm [2]

(b) r = 3.5 cm, h = 7.2 cm [2]

(c) r = 25 mm, h = 10 cm [2]

(d) r = 0.3 cm, h = 17 mm [2]

27.8.3 Calculate the volume and total surface area of each of the following right-angled triangular prisms.

(a) (The triangular end has the right angle between sides of 4 cm and 3 cm, and a sloping side of 5 cm. The prism is 8 cm long.) [4]

(b) (The triangular end is isosceles with two equal sides of 5.0 cm meeting at a right angle, and a base of 7.1 cm. The prism is 8.2 cm long.) [4]

27.8.4 Calculate the volume of each of the following prisms. All dimensions are given in centimetres.

(a) (The end face is a 4 by 4 square with a slot 2 wide and 1 high cut out of the middle of its bottom edge, leaving 1 on each side. The prism is 5 long.) [2]

(b) (A staircase of three steps. Each step is 1 deep and 2 high, so the end face is made of columns 1 wide and 6, 4 and 2 high. The prism is 8 long.) [2]

(c) (The end face is a house shape: a rectangle 5 wide and 7 tall with a triangle on top, making the total height 12. The prism is 8 long.) [2]

(d) (The end face is a trapezium with parallel sides 7 (top) and 4 (bottom), 5 apart. The prism is 5 long.) [2]

Exercise 27.9 · Prism problems

27.9.1 The diagram shows a plan view of a cylinder inside a box the shape of a cube. If the radius of the cylinder is 8 cm, calculate the percentage volume of the cube not occupied by the cylinder. (The cylinder touches all four sides of the box and is as tall as the box.) [4]

27.9.2 A chocolate bar is made in the shape of a triangular prism. The triangular face of the prism is equilateral and has an edge length of 4 cm and a perpendicular height of 3.5 cm. The manufacturer also sells these in special packs of six bars arranged as a hexagonal prism. If the prisms are 20 cm long, calculate:

(a) the cross-sectional area of the pack, [2]

(b) the volume of the pack. [2]

27.9.3 A cuboid and a cylinder have the same volume. The radius and height of the cylinder are 2.5 cm and 8 cm respectively. If the length and width of the cuboid are each 5 cm, calculate its height to 1 d.p. [3]

27.9.4 A section of steel pipe is shown in the diagram. The inner radius is 35 cm and the outer radius 36 cm. Calculate the volume of steel used in making the pipe if it has a length of 130 m. Give your answer in terms of π. [4]

Exercise 27.10 · Arc length

27.10.1 For each of the following, give the length of the arc to 3 s.f. O is the centre of the circle.

(a) 45°, r = 8 cm	(b) 8°, r = 15 cm	(c) 110°, r = 6 cm	(d) 270°, r = 5 cm

27.10.2 A sector is the region of a circle enclosed by two radii and an arc. Calculate the angle φ for each of the following sectors. The radius r and arc length a are given in each case.

(a) r = 14 cm, a = 8 cm	(b) r = 4 cm, a = 16 cm	(c) r = 7.5 cm, a = 7.5 cm	(d) r = 6.8 cm, a = 13.6 cm

27.10.3 Calculate the radius r for each of the following sectors. The angle φ and arc length a are given in each case.

(a) φ = 75°, a = 16 cm	(b) φ = 300°, a = 24 cm	(c) φ = 20°, a = 6.5 cm	(d) φ = 243°, a = 17 cm

Exercise 27.11 · Perimeters with arcs

27.11.1 Calculate the perimeter of each of these shapes. Give your answers in exact form.

(a) (Two sectors meet at their shared centre O, opposite each other. Both have an angle of 50°. One has radius 12 cm and the other radius 5 cm.) [3]

(b) (A quarter circle of radius 24 cm and a quarter circle of radius 12 cm share centre O and stand side by side on a straight base of 36 cm. A vertical 12 cm edge joins the top of the large arc to the top of the small arc.) [3]

27.11.2 A shape is made from two sectors arranged in such a way that they share the same centre. The radius of the smaller sector is 7 cm and the radius of the larger sector is 10 cm. If the angle at the centre of the smaller sector is 30° and the arc length of the larger sector is 12 cm, calculate:

(a) the arc length of the smaller sector, [2]

(b) the total perimeter of the two sectors, [2]

(c) the angle at the centre of the larger sector. [2]

27.11.3 For the diagram (right), calculate: (A sector of radius 24 cm and arc length 20 cm, centre O and angle φ. A smaller sector with the same centre and angle, arc length 10 cm, is cut out of it.)

(a) the radius of the smaller sector, [2]

(b) the perimeter of the shape, [2]

(c) the angle φ. [2]

Exercise 27.12 · The area of a sector

27.12.1 Calculate the area of each of the following sectors, using the values of the angle φ and radius r in each case.

(a) φ = 60°, r = 8 cm	(b) φ = 120°, r = 14 cm	(c) φ = 2°, r = 18 cm	(d) φ = 320°, r = 4 cm

27.12.2 Calculate the radius for each of the following sectors, using the values of the angle φ and the area A in each case.

(a) φ = 40°, A = 120 cm^2	(b) φ = 12°, A = 42 cm^2	(c) φ = 150°, A = 4 cm^2	(d) φ = 300°, A = 400 cm^2

27.12.3 Calculate the value of the angle φ, to the nearest degree, for each of the following sectors, using the values of A and r in each case.

(a) r = 12 cm, A = 60 cm^2	(b) r = 26 cm, A = 0.02 m^2	(c) r = 0.32 m, A = 180 cm^2	(d) r = 38 mm, A = 16 cm^2

Exercise 27.13 · Sector problems

27.13.1 A rotating sprinkler is placed in one corner of a garden (below). If it has a reach of 8 m and rotates through an angle of 30°, calculate the area of garden not being watered. Give your answer in terms of π. (The garden is a rectangle 12 m by 8 m.) [3]

27.13.2 Two sectors AOB and COD share the same centre O. The area of AOB is three times the area of COD. Calculate: (Both sectors have an angle of 60° at O. OA = 15 cm and OC = r cm, with C on OA and D on OB.)

(a) the area of sector AOB, [2]

(b) the area of sector COD, [1]

(c) the radius r cm of sector COD. [2]

27.13.3 A circular cake is cut. One of the slices is shown. Calculate: (The slice has a sector-shaped top with radius 10 cm and angle 24°, and it is 3 cm high. The curved edge of the top is a cm long.)

(a) the length a cm of the arc, [2]

(b) the total surface area of all the sides of the slice, [3]

(c) the volume of the slice. [2]

27.13.4 The diagram shows a plan view of four tiles in the shape of sectors placed in the bottom of a box. C is the midpoint of the arc AB and intersects the chord AB at point D. If the length OB is 10 cm, calculate: (Each tile is a sector of radius 10 cm with angle 40° at its centre. The tiles fit together in a rectangular box; OC is the line of symmetry of sector AOB and crosses the chord AB at D.)

(a) the length OD, [2]

(b) the length CD, [1]

(c) the area of the sector AOB, [2]

(d) the length and width of the box, [3]

(e) the area of the base of the box not covered by the tiles. [2]

27.13.5 The tiles in Question 4 are repackaged and are now placed in a box, the base of which is a parallelogram. Given that C and F are the midpoints of arcs AB and OG respectively, calculate: (The same four tiles, arranged in a parallelogram-shaped box. E lies on CF, and the 40° angle is at O.)

(a) the angle OCF, [2]

(b) the length CE, [2]

(c) the length of the sides of the box, [3]

(d) the area of the base of the box not covered by the tiles. [2]

Exercise 27.14 · The volume of a sphere

27.14.1 Calculate the volume of each of the following spheres. The radius r is given in each case.

(a) r = 6 cm	(b) r = 9.5 cm	(c) r = 8.2 cm	(d) r = 0.7 cm

27.14.2 Calculate the radius of each of the following spheres. Give your answers in centimetres and to 1 d.p. The volume V is given in each case.

(a) V = 130 cm^3	(b) V = 720 cm^3	(c) V = 0.2 m^3	(d) V = 1000 mm^3

Exercise 27.15 · Sphere problems

27.15.1 Given that sphere B has twice the volume of sphere A, calculate the radius of sphere B. Give your answer to 1 d.p. (Sphere A has radius 5 cm; sphere B has radius r cm.) [3]

27.15.2 Calculate the volume of material used to make the hemispherical bowl if the inner radius of the bowl is 5 cm and its outer radius 5.5 cm. Give your answer in terms of π. [3]

27.15.3 The volume of the material used to make the sphere and hemispherical bowl are the same. Given that the radius of the sphere is 7 cm and the inner radius of the bowl is 10 cm, calculate, to 1 d.p., the outer radius r cm of the bowl. [4]

27.15.4 A ball is placed inside a box into which it will fit tightly. If the radius of the ball is 10 cm, calculate the percentage volume of the box not occupied by the ball. (The box is a cube.) [4]

27.15.5 A steel ball is melted down to make eight smaller identical balls. If the radius of the original steel ball was 20 cm, calculate to the nearest millimetre the radius of each of the smaller balls. [3]

27.15.6 A steel ball of volume 600 cm^3 is melted down and made into three smaller balls A, B and C. If the volumes of A, B and C are in the ratio 7 : 5 : 3, calculate to 1 d.p. the radius of each of A, B and C. [5]

27.15.7 The cylinder and sphere shown have the same radius and the same height. Calculate the ratio of their volumes, giving your answer in the form volume of cylinder : volume of sphere. (Both have radius r.) [3]

Exercise 27.16 · The surface area of a sphere

27.16.1 Calculate the surface area of each of the following spheres when:

(a) r = 6 cm	(b) r = 4.5 cm	(c) r = 12.25 cm	(d) r = 1/π cm

27.16.2 Calculate the radius of each of the following spheres, given the surface area in each case.

(a) A = 50 cm^2	(b) A = 16.5 cm^2	(c) A = 120 mm^2	(d) A = π cm^2

27.16.3 Sphere A has a radius of 8 cm and sphere B has a radius of 16 cm. Calculate the ratio of their surface areas in the form 1 : n. [2]

27.16.4 A hemisphere of diameter 10 cm is attached to a cylinder of equal diameter as shown. If the total length of the shape is 20 cm, calculate the surface area of the whole shape. [4]

27.16.5 A sphere and a cylinder both have the same surface area and the same height of 16 cm. Calculate the radius of the cylinder. [4]

Exercise 27.17 · The volume of a pyramid

27.17.0 Find the volume of each of the following pyramids:

(1) (A rectangular base 5 cm by 4 cm, perpendicular height 6 cm.) [2]

(2) (Base area = 50 cm^2, perpendicular height 8 cm.) [2]

(3) (A right-angled triangular base with sides 6 cm and 10 cm next to the right angle. The apex is 8 cm straight above the right angle.) [2]

(4) (A rectangular base 7 cm by 6 cm, perpendicular height 5 cm.) [2]

Exercise 27.18 · Pyramid problems

27.18.1 Calculate the perpendicular height h cm for the pyramid, given that it has a volume of 168 cm^3. (A rectangular base 9 cm by 8 cm.) [2]

27.18.2 Calculate the length of the edge marked x cm, given that the volume of the pyramid is 14 cm^3. (A right-angled triangular base with sides 3 cm and x cm next to the right angle. The apex is 7 cm straight above the right angle.) [3]

27.18.3 The top of a square-based pyramid (right) is cut off. The cut is made parallel to the base. If the base of the smaller pyramid has a side length of 3 cm and the vertical height of the truncated pyramid is 6 cm, calculate: (The original base is a 12 cm by 12 cm square.)

(a) the height of the original pyramid, [3]

(b) the volume of the original pyramid, [2]

(c) the volume of the truncated pyramid. [2]

27.18.4 The top of a triangular-based pyramid (tetrahedron) is cut off. The cut is made parallel to the base. If the vertical height of the top is 6 cm, calculate: (The book's diagram shows the small top pyramid with a vertical height of 6 cm and a base edge of 5 cm, and the truncated piece below it. No other lengths are printed, so use any measurements your teacher gives you.)

(a) the height of the truncated piece, [2]

(b) the volume of the small pyramid, [2]

(c) the volume of the original pyramid. [2]

Exercise 27.19 · The surface area of a pyramid

27.19.1 Calculate the surface area of a regular tetrahedron with edge length 2 cm. [3]

27.19.2 The rectangular-based pyramid shown has a sloping edge length of 12 cm. Calculate its surface area. (The base is 8 cm by 5 cm.) [4]

27.19.3 Two square-based pyramids are glued together as shown (right). Given that all the triangular faces are congruent, calculate the surface area of the whole shape. (The square base has side 4 cm and each sloping edge is 5 cm.) [4]

27.19.4 Calculate the surface area of the truncated square-based pyramid shown. Assume that all the sloping faces are congruent. (The top square has side 9 cm, the bottom square has side 18 cm and each sloping edge is 14 cm.) [5]

27.19.5 The two pyramids shown below have the same surface area. Calculate: (A regular tetrahedron with edge 20 cm, and a square-based pyramid with base side 12 cm and sloping edge x cm.)

(a) the surface area of the regular tetrahedron, [3]

(b) the area of one of the triangular faces on the square-based pyramid, [2]

(c) the value of x. [3]

Exercise 27.20 · The volume of a cone

27.20.1 Calculate the volume of each of the following cones. Use the values for the base radius r and the vertical height h given in each case.

(a) r = 3 cm, h = 6 cm	(b) r = 6 cm, h = 7 cm	(c) r = 8 mm, h = 2 cm	(d) r = 6 cm, h = 44 mm

27.20.2 Calculate the base radius of each of the following cones. Use the values for the volume V and the vertical height h given in each case.

(a) V = 600 cm^3, h = 12 cm	(b) V = 225 cm^3, h = 18 mm	(c) V = 1400 mm^3, h = 2 cm	(d) V = 0.04 m^3, h = 145 mm

27.20.3 The base circumference C and the length of the sloping face l are given for each of the following cones. Calculate (i) the base radius, (ii) the vertical height, (iii) the volume in each case. Give all answers to 3 s.f.

(a) C = 50 cm, l = 15 cm [4]

(b) C = 100 cm, l = 18 cm [4]

(c) C = 0.4 m, l = 75 mm [4]

(d) C = 240 mm, l = 6 cm [4]

Exercise 27.21 · Cone problems

27.21.1 The two cones A and B shown below have the same volume. Using the dimensions shown and given that the base circumference of cone B is 60 cm, calculate the height h cm. (Cone A has base radius 5 cm and sloping face 15 cm.) [5]

27.21.2 The sector shown is assembled to form a cone. Calculate the following, giving your answers to parts a, b and c in exact form: (The sector has radius 9 cm and angle 210°.)

(a) the base circumference of the cone, [2]

(b) the base radius of the cone, [2]

(c) the vertical height of the cone, [2]

(d) the volume of the cone. Give your answer correct to 3 s.f. [2]

27.21.3 A cone is placed inside a cuboid as shown. If the base diameter of the cone is 12 cm and the height of the cuboid is 16 cm, calculate the volume of the cuboid not occupied by the cone. (The cone fits exactly: the base of the cuboid is a 12 cm square and the cone is as tall as the cuboid.) [4]

27.21.4 Two similar sectors are assembled into cones (below). Calculate the ratio of their volumes. (Both sectors have an angle of 260°. Their radii are 6 cm and 9 cm.) [4]

Exercise 27.22 · Compound solids

27.22.1 An ice cream consists of a hemisphere and a cone (right). Calculate, in exact form, its total volume. (The hemisphere and cone share a radius of 2.5 cm. The total height from the tip of the cone to the top of the hemisphere is 10 cm.) [4]

27.22.2 A cone is placed on top of a cylinder. Using the dimensions given (right), calculate the total volume of the shape. (Both have a base diameter of 8 m. The cone is 10 m tall and the cylinder is 12 m tall.) [4]

27.22.3 Two identical truncated cones are placed end to end as shown. Calculate the total volume of the shape. (The shape is 36 cm long. Each wide end has a diameter of 16 cm and the narrow middle, where the two meet, has a diameter of 4 cm.) [5]

27.22.4 Two cones A and B are placed either end of a cylindrical tube as shown. Given that the volumes of A and B are in the ratio 2 : 1, calculate: (The tube is 25 cm long with a diameter of 10 cm. Each cone has its base at one end of the tube and the two apexes meet inside it.)

(a) the volume of cone A, [2]

(b) the height of cone B, [2]

(c) the volume of the cylinder. [2]

Exercise 27.23 · The surface area of a cone

27.23.1 Calculate the surface area of each of the following cones:

(a) (Base radius 6 cm, sloping face 16 cm.) [3]

(b) (Base radius 15 cm, sloping face 20 cm.) [3]

27.23.2 Two cones with the same base radius are stuck together as shown. Calculate the surface area of the shape. (The shared base radius is 8 cm. The sloping faces are 15 cm and 30 cm.) [4]

Student assessment 1

SA1.1 Calculate the area of the shape below. (A rectangle 15 cm long and 4 cm tall with a triangle on top. The base of the triangle is the 15 cm top of the rectangle, and the total height of the shape is 8 cm.) [3]

SA1.2 Calculate the circumference and area of each of the following circles. Give your answers to 3 s.f.

(a) (Radius 5.5 cm.) [2]

(b) (Diameter 16 mm.) [2]

SA1.3 A semicircular shape is cut out of the side of a rectangle as shown. Calculate the shaded area, giving your answer in exact form. (The rectangle is 6 cm wide and 4 cm tall. The semicircle has the 6 cm top edge as its diameter.) [3]

SA1.4 For the diagram (right), calculate the area of: (From the top: a parallelogram with a 7 cm top side and height 4 cm; then a trapezium of height 4 cm whose top is 7 cm and whose bottom is the diameter of the semicircle; then a semicircle of radius 5 cm.)

(a) the semicircle, [2]

(b) the trapezium, [2]

(c) the whole shape. [2]

SA1.5 A cylindrical tube has an inner diameter of 6 cm, an outer diameter of 7 cm and a length of 15 cm. Calculate the following to 3 s.f.:

(a) the surface area of the shaded end, [2]

(b) the inside surface area of the tube, [2]

(c) the total surface area of the tube. [3]

SA1.6 Calculate the volume of each of the following cylinders. Give your answers in terms of π.

(a) (Radius 3 mm, height 12 mm.) [2]

(b) (Diameter 2.5 cm, height 2 cm.) [2]

Student assessment 2

SA2.1 Calculate the arc length of each of the following sectors. The angle φ and radius r are given in each case.

(a) φ = 45°, r = 15 cm [2]

(b) $\varphi = 150°$, $r = 13.5$ cm [2]

SA2.2 Calculate the angle φ in each of the following sectors. The radius r and arc length a are given in each case.

(a) $r = 20$ mm, $a = 95$ mm [2]

(b) $r = 9$ cm, $a = 9$ mm [2]

SA2.3 Calculate the area of the sector shown, giving your answer in exact form. (Radius 18 cm, angle 40°.) [2]

SA2.4 A sphere has a radius of 6.5 cm. Calculate to 3 s.f:

(a) its total surface area, [2]

(b) its volume. [2]

SA2.5 A pyramid with a base the shape of a regular hexagon is shown. If the length of each of its sloping edges is 24 cm, calculate: (Each side of the hexagonal base is 12 cm.)

(a) its total surface area, [5]

(b) its volume. [4]

Student assessment 3

SA3.1 The prism has a cross-sectional area in the shape of a sector. Calculate: (The sector has angle 50°, radius r cm and arc length 20 cm. The prism is 8 cm long.)

(a) the radius r cm, [2]

(b) the cross-sectional area of the prism, [2]

(c) the total surface area of the prism, [3]

(d) the volume of the prism. [2]

SA3.2 The cone and sphere shown (below) have the same volume. If the radius of the sphere and the height of the cone are both 6 cm, calculate each of the following, give your answers in exact form:

(a) the volume of the sphere, [2]

(b) the base radius of the cone, [2]

(c) the slant height x cm, [2]

(d) the surface area of the cone. [3]

SA3.3 The top of a cone is cut off and a cylindrical hole is drilled out of the remaining truncated cone as shown (right). Calculate: (The cone's base diameter is 56 mm. The truncated cone is 50 mm tall, and its flat top has a diameter of 28 mm, the same as the cylindrical hole, which goes all the way through.)

(a) the height of the original cone, [2]

(b) the volume of the original cone, [2]

(c) the volume of the solid truncated cone, [2]

(d) the volume of the cylindrical hole, [2]

(e) the volume of the remaining truncated cone. [1]

Check yourself

1. Explain why the area of a triangle is half of base × height. Draw a sketch to help. [2]

2. A semicircle has diameter 10 cm. A student says its perimeter is 5π cm. Explain the mistake and give the correct perimeter in exact form. [3]

3. A sector has radius 6 cm and angle 120°. Find its arc length and its area, both in terms of π. [4]

4 A cylinder and a cone have the same radius and the same height. How many of the cones would fill the cylinder? Explain using the formulae. [2]

5 A solid hemisphere has radius 3 cm. Find its total surface area in terms of π, and say why it is not just half of $4\pi r^2$. [3]

Chapter quiz

Q1 A triangle has base 10 cm and perpendicular height 12 cm. What is its area?

A	B	C	D
120 cm²	60 cm²	22 cm²	30 cm²

Q2 A trapezium has parallel sides 13 cm and 8 cm, 8 cm apart. What is its area?

A	B	C	D
104 cm²	84 cm²	168 cm²	42 cm²

Q3 What is the circumference of a circle of radius 3.5 cm, to 3 significant figures?

A	B	C	D
11.0 cm	22.0 cm	38.5 cm	7.00 cm

Q4 A circle has diameter 10 cm. What is its area in exact form?

A	B	C	D
100π cm²	25π cm²	10π cm²	5π cm²

Q5 What is the length of an arc of angle 90° in a circle of radius 8 cm?

A	B	C	D
2π cm	4π cm	8π cm	16π cm

Q6 What is the area of a sector of angle 60° and radius 6 cm?

A	B	C	D
3π cm²	6π cm²	12π cm²	36π cm²

Q7 A cuboid measures 4 cm by 6 cm by 8 cm. What is its total surface area?

A	B	C	D
104 cm²	192 cm²	208 cm²	384 cm²

Q8 A cylinder has radius 3 cm and height 7 cm. What is its volume?

A	B	C	D
21π cm³	42π cm³	63π cm³	126π cm³

Q9 What is the volume of a sphere of radius 3 cm?

A	B	C	D
12π cm³	27π cm³	36π cm³	108π cm³

Q10 A square-based pyramid has base side 6 cm and perpendicular height 5 cm. Its volume is:

A	B	C	D
180 cm³	30 cm³	60 cm³	90 cm³

Q11 A cone has base radius 3 cm and vertical height 4 cm. What is its volume?

A	B	C	D
12π cm³	36π cm³	48π cm³	15π cm³

Q12 A cone and a cylinder have the same radius and height. The cone holds what fraction?

A	B	C	D
½ of the cylinder	⅓ of the cylinder	¼ of the cylinder	⅔ of the cylinder

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End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-27-perimeter-area-and-volume/