C25

Angle properties

CHAPTER ASSIGNMENT · 13 MARKS

Name _____ Class _____ No. _____ Date _____

Show your reasoning for every angle you find, for example "angles on a straight line add up to 180°". All diagrams are not drawn to scale. Calculate the angles; do not measure them.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 25 (Angle properties), for classroom use. The book's diagrams are described in words in brackets; where a description is not enough, use the book's figure. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 25.1 · Angles at a point, on a line and in parallel lines

25.1.1 In each of the following questions, some of the angles are given. Deduce, giving your reasons, the size of the other labelled angles.

(Diagram: three lines cross at one point. Around the point are four marked angles: p°, q°, q° (the two q° angles are equal and next to each other) and a 126° angle. Together they fill the point.)

Find p and q.

25.1.2 (Diagram: lines cross at one point. Around the point are five marked angles: 45°, 80°, c°, b° and a°.)

Find a, b and c.

25.1.3 (Diagram: two parallel horizontal lines, marked with arrows, are crossed by a sloping line. At the upper crossing the angles are y° (above the line, left of the sloping line), x° (below the line, left) and z° (below the line, right). At the lower crossing the angles are w° (above the line, left), v° (below the line, left) and 60° (below the line, right).)

Find v, w, x, y and z.

25.1.4 (Diagram: a triangle with a line drawn across it parallel to its base, both marked with arrows. The base angles are 50° at the left and c° at the right. On the left side, a° is above the parallel line and b° is below it. On the right side, 45° is above the parallel line and d° is below it.)

Find a, b, c and d.

25.1.5 (Diagram: two horizontal parallel lines, marked with double arrows, crossed by two sloping parallel lines, marked with single arrows. The angles marked are 45° at the right-hand sloping line where it meets the upper horizontal line, r° where the left-hand sloping line meets the upper horizontal line, s° and t° where the left-hand sloping line meets the lower horizontal line, and q° and p° where the right-hand sloping line meets the lower horizontal line.)

Find p, q, r, s and t.

25.1.6 (Diagram: two horizontal parallel lines, marked with arrows, joined by two lines that cross each other between them to make a bow-tie shape. The top line has d° at its left end and 30° at its right end. The bottom line has e° at its left end and 70° at its right end.)

Find d and e.

25.1.7 (Diagram: three lines meet at a point. The marked angles are a°, 2a° and a right angle (marked with a small square) on one side, and a 159° angle.)

Find the value of a.

25.1.8 (Diagram: a sloping line crosses two parallel lines, marked with arrows. At the upper crossing, 3a° is the angle below the upper line on the right of the sloping line. At the lower crossing, 2a° is the angle above the lower line on the right of the sloping line.)

Find the value of a.

Exercise 25.2 · Angles in a triangle

25.2.1 For each of the triangles below, use the information given to calculate the size of angle x.

(a) (Diagram: an isosceles triangle with its two equal sides marked. It has a vertex at the left and a vertical side on the right. The angle at the top right is x° and the angle at the bottom right is 70°.)

(b) (Diagram: a triangle with angles 95° at the left vertex, 30° at the top right vertex and x° at the bottom vertex.)

(c) (Diagram: an equilateral triangle, all three sides marked equal, with x° at the bottom vertex.)

(d) (Diagram: a triangle with angles 42° at the top, 65° at the left and x° at the bottom right.)

(e) (Diagram: a triangle with a right angle at the top left. The vertical side and the top side are marked equal. x° is the angle at the bottom.)

(f) (Diagram: an isosceles triangle with its two sloping sides marked equal. The angle at the left is 35° and x° is the angle at the top.)

25.2.2 In each of the diagrams below, calculate the size of the labelled angles.

(a) (Diagram: a triangle with 60° at the bottom left and 45° at the bottom right. A line from the top vertex meets the base at a right angle. It splits the top angle into a° on the left and b° on the right.)

(b) (Diagram: a triangle with a line from its bottom-left vertex to the opposite side, making two triangles. The labelled angles are x°, 100°, y°, 30° and z° around the left-hand part, and 30° at the bottom right vertex.)

(c) (Diagram: a triangle with a line drawn from the bottom-left vertex to the right-hand vertex area. The labelled angles are 70°, q°, 35°, 50°, p° and r°.)

(d) (Diagram: an isosceles triangle with equal sides marked, split in half by a line from the top vertex to the middle of the base, at a right angle. The angles marked are 35° and d° at the top, and f° and e° at the base.)

(e) (Diagram: a thin triangle with a line from a vertex to the opposite side. Equal sides are marked with ticks. The labelled angles are d°, c°, 125°, e°, b° and a°.)

(f) (Diagram: a right-angled triangle with the right angle at the bottom left. The vertical side and the long sloping side have ticks marking equal sides. A line from the bottom-left corner meets the sloping side. The labelled angles are p° at the top, r° and s° where the line meets the sloping side, and q° at the bottom right.)

Exercise 25.3 · Angles in a quadrilateral

25.3.1 In each of the diagrams below, calculate the size of the labelled angles.

(Diagram: a quadrilateral with angles a°, 100°, 70° and 75°.)

25.3.2 (Diagram: a four-sided shape with angles x° at the left, y° at the top, 40° at the right and z° at the bottom.)

25.3.3 (Diagram: a kite with its pairs of equal sides marked. The angles are m° at the top, 85° at the left, 125° at the right and n° at the bottom.)

25.3.4 (Diagram: a four-sided shape with some sides marked equal and arrows on two sides. The angles are 65° at the top left, u° and t° at the right and s° at the bottom left.)

25.3.5 (Diagram: a parallelogram, with arrows on its parallel sides. The angles are j° at the top left, i° at the top right, k° at the bottom left and h° at the bottom right. A 60° angle is marked outside the shape at the bottom right.)

25.3.6 (Diagram: a trapezium with a diagonal drawn inside it. The labelled angles are e° and c° at the top, a° at the top right, d° and b° at the bottom left, and 80° at the bottom right.)

25.3.7 (Diagram: a four-sided shape with a right angle at the bottom left. A diagonal is drawn. The labelled angles are 45° and p° at the top left, 50° and q° and r° at the bottom right.)

25.3.8 (Diagram: a four-sided shape divided into triangles by two lines. The labelled angles are v°, r°, 50°, p°, u°, t°, s°, q° and 75°.)

Exercise 25.4 · Polygons

25.4.1 Find the sum of the interior angles of the following polygons:

(a) a hexagon

(b) a nonagon

(c) a heptagon

25.4.2 Find the value of each interior angle of the following regular polygons:

(a) an octagon

(b) a square

(c) a decagon

(d) a dodecagon

25.4.3 Find the size of each exterior angle of the following regular polygons:

(a) a pentagon

(b) a dodecagon

(c) a heptagon

25.4.4 The exterior angles of regular polygons are given below. In each case calculate the number of sides the polygon has.

(a) 20°

(b) 36°

(c) 10°

(d) 45°

(e) 18°

(f) 3°

25.4.5 The interior angles of regular polygons are given below. In each case calculate the number of sides the polygon has.

(a) 108°

(b) 150°

(c) 162°

(d) 156°

(e) 171°

(f) 179°

25.4.6 Calculate the number of sides a regular polygon has if an interior angle is five times the size of an exterior angle.

Exercise 25.5 · The angle in a semicircle

25.5.1 In each of the following diagrams, O marks the centre of the circle. Calculate the value of x in each case.

(Diagram: a triangle drawn inside a circle. One side of the triangle is a diameter through O. The angle at one end of the diameter is 30° and the angle x° is at the other vertex end, at the lower left.)

25.5.2 (Diagram: a triangle inside a circle with the diameter as its top side. The angle at the left end of the diameter is 45°. The side from the right end of the diameter is extended outside the circle, and x° is the angle outside the circle at the right end, between the extended line and the diameter.)

25.5.3 (Diagram: a triangle inside a circle. One side passes through O. A line is extended outside the circle at the left, making an angle of 110° with the triangle's side. x° is the angle at the top right of the triangle.)

25.5.4 (Diagram: a rectangle-like four-sided shape drawn inside a circle, with a diagonal through O. Arrows mark one pair of parallel sides. The angle between the diagonal and one side is 58°, and x° is the angle next to it at the same vertex.)

25.5.5 (Diagram: a triangle inside a circle with the diameter as its top side. A line from the left end of the diameter makes a 20° angle with the diameter. x° is the angle at the vertex marked inside the shape, near O, between that line and the diameter's line to the lower vertex.)

25.5.6 (Diagram: a circle with centre O, and a right-angled triangle whose right angle is at the bottom right. Its horizontal side is a diameter. The vertical side and the long sloping side are marked equal. x° is the angle at the top.)

Exercise 25.6 · The angle between a tangent and a radius

25.6.1 In each of the following diagrams, O marks the centre of the circle. Calculate the value of x in each case.

(Diagram: a circle with a vertical tangent on its right. A radius goes from O to the point of contact. A line from O makes an angle of 55° with that radius and meets the tangent. x° is the angle where that line meets the tangent.)

25.6.2 (Diagram: a circle with two tangents drawn from a point below it. x° is the angle at O between the two radii to the points of contact, as shown. The angle at the point below the circle, between the tangents, is 30°.)

25.6.3 (Diagram: a circle with two tangents meeting at a point at the lower left. The angle at O between the two radii to the points of contact is 140°. x° is the angle between the tangents at the lower left.)

25.6.4 (Diagram: a circle with a vertical tangent on its right. A horizontal diameter ends at the point of contact. A line from the other end of the diameter goes up to meet the tangent. The tangent segment and the diameter are marked equal. x° is the angle at the top between the tangent and the sloping line.)

25.6.5 (Diagram: a circle with two tangents meeting below it. The angle between the tangents is 24°. x° is the angle at the centre O between the line from O to one point of contact and the line joining the two points of contact.)

25.6.6 (Diagram: a circle with a tangent and a long line through O. A triangle is formed. The angle where the lines meet below the circle is 38°, and x° is marked inside the circle near the tangent.)

25.6.7 (Diagram: a circle with centre O and a tangent. O is joined to a point on the circle with a radius of 5 cm. The tangent from that point has length 12 cm, and the line from O to the end of the tangent is x cm.)

Calculate x.

25.6.8 (Diagram: a circle with centre O and a tangent. The radius is 6 cm. The distance from O to a point outside the circle along the tangent line is 10 cm, and the tangent length is x cm.)

Calculate x.

25.6.9 (Diagram: a circle with centre O and two tangents from an outside point. The radius is 5 cm. Each tangent has length 17 cm, and x cm is the distance from O to the outside point.)

Calculate x.

Exercise 25.7 · Irregular polygons

25.7.1 For the pentagon:

(Diagram: a pentagon with interior angles 2x°, 3x°, x°, 3x° and x°.)

(a) calculate the value of x,

(b) calculate the size of each of the angles.

25.7.2 Find the size of each angle in the octagon (below).

(Diagram: an octagon with four angles marked x° and four angles marked (x + 20)°. Starting at the top and going clockwise, the angles are (x + 20)°, (x + 20)°, x°, x°, (x + 20)°, (x + 20)°, x°, x°.)

25.7.3 Calculate the value of x for the pentagon shown.

(Diagram: a pentagon with one side extended at each vertex to show the exterior angles. The exterior angles are x°, x°, (x + 20)°, (x + 20)° and x°.)

25.7.4 Calculate the size of each of the angles a, b, c, d and e in the hexagon.

(Diagram: a hexagon with each side extended to show the exterior angles. The exterior angles are x°, x°, (x + 50)°, (x + 25)°, (x + 5)° and (x + 10)°. The interior angles are a°, a°, e°, d°, c° and b°.)

Exercise 25.8 · Angle at the centre of a circle

25.8.1 In each of the following diagrams, O marks the centre of the circle. Calculate the size of the marked angles:

(Diagram: an angle of x° at the circumference and an angle of 110° at the centre O, both standing on the same arc.)

25.8.2 (Diagram: an angle of x° at O, and an angle of 140° outside the circle where an extended line meets the circle.)

25.8.3 (Diagram: an angle of x° at the circumference, and an angle of 180° at the centre O, on the same arc.)

25.8.4 (Diagram: an angle of 130° at the circumference, and an angle of x° at the centre O.)

25.8.5 (Diagram: an angle of x° at the centre O, and an angle of 40° at the circumference.)

25.8.6 (Diagram: an angle of 100° at the circumference at the top, and an angle of x° at the centre O.)

25.8.7 (Diagram: an isosceles triangle inscribed in a circle, with its two equal sides marked. The angle at the top is 36°. O is the centre. y° is the angle at the circumference on the right, and x° is the angle at the bottom left.)

25.8.8 (Diagram: an isosceles triangle inscribed in a circle, with its two equal sides marked. O is the centre. x° is the angle at the top, y° is the angle at the circumference on the right, and the angle at the bottom left is 40°.)

Exercise 25.9 · Angles in the same segment

25.9.1 Calculate the marked angles in the following diagrams:

(Diagram: a triangle inside a circle with two chords crossing inside it. One angle at the circumference is 72° and another angle on the same arc is a°.)

25.9.2 (Diagram: two chords through O forming a bow-tie shape. The angle at the circumference on the left is 33°, b° is the angle at the circumference on the right, and c° is the angle at the centre.)

25.9.3 (Diagram: two chords crossing inside a circle. The angles marked are d° and e° on the left, and 48° and 32° on the right.)

25.9.4 (Diagram: two chords through O forming a bow-tie shape. The angles marked are 30°, f°, g°, h°, i° and j°.)

25.9.5 (Diagram: two chords crossing inside a circle, with an angle of 70° where they cross. Equal chords are marked. The other angles are k°, l°, m° and n°.)

25.9.6 (Diagram: a circle with a line touching it on the left, with an exterior angle of 40°. Two chords cross, with an angle of 75° where they cross. The other angles are p° and q°.)

Exercise 25.10 · Cyclic quadrilaterals and the alternate segment theorem

25.10.1 For each question 1–8, calculate the size of the marked angles.

(Diagram: a cyclic quadrilateral with angles a° at the top left, b° at the right, 115° at the bottom left and 100° at the bottom.)

25.10.2 (Diagram: a cyclic quadrilateral with angles c° at the top left, d° at the top right, 82° at the bottom left, and a 110° angle at the bottom right made with a tangent line. The angle e° is next to the 110° angle.)

25.10.3 (Diagram: a cyclic quadrilateral with angles g° at the top left, f° at the bottom left, and 105° at the bottom right. One side is extended to make an exterior angle of 115° at the top right.)

25.10.4 (Diagram: a cyclic quadrilateral with pairs of equal sides marked. The angles are i° at the top, j° at the left, k° at the right and 82° at the bottom.)

25.10.5 (Diagram: a cyclic quadrilateral with a pair of parallel sides marked by arrows. The angles are m° at the top, 85° at the top right, q° and p° at the left, 30° at the bottom left, l° at the bottom and n° at the bottom right.)

25.10.6 (Diagram: a cyclic quadrilateral with a triangle above it. The angle at the top is 60° and the two sides from it are marked equal. An exterior angle of 10° is shown on the right. The angles of the quadrilateral are s°, u°, t° and r°.)

25.10.7 (Diagram: a cyclic quadrilateral with a diagonal drawn, and a tangent at the lower right. The angles marked are w° at the top, y° at the left, x° at the bottom, v° at the right, 55° between the tangent and a side, and 45° between the tangent and another side.)

25.10.8 (Diagram: a triangle inside a circle with two tangents at two of its vertices, meeting at the lower right. The angle at the left vertex is α, and β is the angle between the tangent and the side at the upper vertex. The angle between the side and the other tangent is 63°. The angle between the tangents is ø.)

Student assessment 1

SA1.1 For the diagrams below, calculate the size of the labelled angles.

(a) (Diagram: two parallel lines, marked with arrows, crossed by a sloping line. The angles are q° and r° at the upper crossing, and 45° and p° at the lower crossing.)

(b) (Diagram: two parallel horizontal lines crossed by two parallel sloping lines. The angles are a° and b° above the upper line, with a 120° angle below it, and c° and d° at the lower line.)

SA1.2 For the diagrams below, calculate the size of the labelled angles.

(a) (Diagram: a right-angled triangle with the right angle at the bottom left and 40° at the bottom right. A line parallel to the base cuts the triangle. The angles are m° at the top, n° and p° on the left and q° on the right.)

(b) (Diagram: a bow-tie made of two triangles that meet at a point. The upper triangle has 55° at the top left and w° at the top right. The angle at the meeting point is 70°. The lower triangle has x° at the meeting point, y° at the lower left and z° at the lower right.)

(c) (Diagram: a trapezium, with parallel sides marked by arrows, overlapped by a triangle. The angles are 80°, d°, c°, e°, 70°, b° and a°.)

SA1.3 Find the size of each interior angle of a twenty-sided regular polygon.

SA1.4 What is the sum of the interior angles of a nonagon?

SA1.5 What is the sum of the exterior angles of a polygon?

SA1.6 What is the size of each exterior angle of a regular pentagon?

SA1.7 If AB is the diameter of the circle and AC = 5 cm and BC = 12 cm, calculate:

(Diagram: a circle with centre O. AB is a diameter. C is a point on the circle, and triangle ABC has AC = 5 cm and BC = 12 cm.)

(a) the size of angle ACB,

(b) the length of the radius of the circle.

In questions 8–11, O marks the centre of the circle. Calculate the size of the angle marked x in each case.

SA1.8 (Diagram: a four-sided shape inside the circle with a diagonal through O. The angle between the diagonal and a side at the circumference is 32°. x° is the angle at the top.)

SA1.9 (Diagram: a triangle with a tangent line. The angle between the line from O and a side of the triangle is 30°. Two sides are marked equal. x° is the angle at the point where the sides meet the circle.)

SA1.10 (Diagram: a circle with centre O and two lines meeting below it. The angle at O is 65° and x° is the angle at the lower vertex.)

SA1.11 (Diagram: a circle with centre O and two tangents meeting at a point below it. The angle at the lower point is 28°, and x° is the angle at O between the two radii.)

SA1.12 (Diagram: a circle with centre O. XY is a tangent to the circle at A. Triangle ABC is inside the circle, and the angle between the tangent and the chord AB is 67°.)

In the diagram above, XY is a tangent to the circle at A. O is the centre of the circle. Calculate each of the following angles, justifying each of your answers.

(a) angle ACB.

(b) The reflex angle AOB.

(c) angle ABO.

Student assessment 2

SA2.1 In the following diagrams, O marks the centre of the circle. Identify which angles are:

(i) supplementary angles,

(ii) right angles,

(iii) equal.

(a) (Diagram: a circle with centre O. A tangent touches the circle at B, and A and C are on the tangent above and below B. OB is a radius.)

(b) (Diagram: a cyclic quadrilateral ABCD with its vertices on a circle with centre O.)

(c) (Diagram: a circle with centre O. Two chords AC and BD cross, and A, B, C and D are on the circle.)

(d) (Diagram: a circle with centre O. CB is a diameter. A is on the circle and the triangle ABC has two equal sides, AC and AB.)

SA2.2 If angle POQ = 84° and O marks the centre of the circle in the diagram, calculate the following:

(Diagram: a circle with centre O and points P, Q and R on the circle. The angle POQ is 84°. OR is equal to OQ, as marked.)

(a) angle PRQ

(b) angle OQR

SA2.3 Calculate angle DAB and angle ABC in the diagram below.

(Diagram: a cyclic quadrilateral ABCD. The angle at D is 88° and the angle at C is 63°.)

SA2.4 If DC is a diameter and O marks the centre of the circle, calculate angles BDC and DAB.

(Diagram: a circle with diameter DC through O. A and B are points on the circle above DC. The angle BCD is 65°.)

SA2.5 Calculate as many angles as possible in the diagram below. O marks the centre of the circle.

(Diagram: two tangents from Q touch the circle at P and R. T is where OQ crosses PR. The angle PQO is 15°.)

SA2.6 Calculate the values of c, d and e.

(Diagram: a circle with two triangles formed by chords. One angle is 35° and another is 54°. The other angles are c°, d° and e°.)

SA2.7 Calculate the values of f, g, h and i.

(Diagram: a cyclic quadrilateral with its sides extended. The angles are h° and g° at the top left, i° at the top right, f° and 85° at the bottom right and 88° at the bottom left.)

SA2.8 (Diagram: a circle through A, B, C and D. XY is a tangent to the circle at A. The angle XAC is 124° and the angle BCA is 73°.)

In the diagram above, XY is a tangent to the circle at A. All four vertices of the quadrilateral ABCD lie on the circumference of the circle.

If angle XAC = 124° and angle BCA = 73°, calculate, giving reasons, the size of the angle BAC.

Check yourself

C1 Find the size of each interior angle of a regular 18-sided polygon. [2]

C2 Each exterior angle of a regular polygon is 24°. How many sides does it have, and what is the sum of its interior angles? [3]

C3 Two angles of a quadrilateral are 90° and 110°. The other two angles are equal. Find them. [2]

C4 A tangent touches a circle with centre O at A. B is on the tangent, and angle AOB is 62°. Find angle ABO, giving a reason. [2]

C5 P, Q and R are on a circle with centre O. The angle PQR on the circumference is 57°. Find the reflex angle POR. [2]

C6 ABCD is a cyclic quadrilateral. Angle ABC = 104°. Find angle ADC, and state the reason. [2]

Chapter quiz

Q1 What is the sum of the interior angles of a hexagon?

A	B	C	D
540°	720°	900°	1080°

Q2 What is each exterior angle of a regular polygon with 20 sides?

A	B	C	D
9°	18°	20°	162°

Q3 Each exterior angle of a regular polygon is 40°. How many sides does it have?

A	B	C	D
8	9	10	36

Q4 Two angles of a triangle are 47° and 85°. What is the third angle?

A	B	C	D
38°	48°	132°	228°

Q5 A quadrilateral has angles x, 2x, 3x and 4x. What is the largest angle?

A	B	C	D
120°	144°	90°	160°

Q6 AB is a diameter of a circle and C is on the circumference. What is angle ACB?

A	B	C	D
45°	60°	90°	180°

Q7 An angle of 130° is at the centre of a circle. What is the angle on the circumference on the same arc?

A	B	C	D
65°	130°	260°	50°

Q8 ABCD is a cyclic quadrilateral and angle A = 78°. What is the opposite angle C?

A	B	C	D
78°	102°	12°	282°

Q9 The tangent-chord angle at A is 52°. What is the angle in the alternate segment?

A	B	C	D
38°	52°	104°	128°

Q10 A radius meets a tangent at the point of contact. What is the angle between them?

A	B	C	D
45°	60°	90°	180°

Q11 Each interior angle of a regular polygon is 150°. How many sides does it have?

A	B	C	D
10	12	15	24

Q12 A line crosses two parallel lines. Angles in a Z shape between the lines are called what?

A	B	C	D
Corresponding angles	Alternate angles	Vertically opposite	Supplementary angles

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End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-25-angle-properties/