C23

Similarity and congruence

CHAPTER ASSIGNMENT · 180 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Show your working. NB: All diagrams are not drawn to scale. You may use a calculator; give answers to 3 significant figures where they are not exact.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 23 (Similarity and congruence), for classroom use. Questions that used a diagram in the book describe it in words (in brackets) or give its measurements as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 23.1 · Similar shapes

23.1.1 (Diagram: the first triangle is right-angled, with sides 6 cm and 8 cm either side of the right angle, hypotenuse 10 cm, and an angle of 60° between the 6 cm side and the 10 cm side. The second triangle is right-angled, with hypotenuse x cm, a short side y cm and a side of 5 cm; the angle between the x cm side and the 5 cm side is 30°.)

(a) Explain why the two triangles (below) are similar. [2]

(b) Calculate the scale factor which reduces the larger triangle to the smaller one. [1]

(c) Calculate the value of x and the value of y. [2]

23.1.2 Which of the triangles below are similar? [3]

(The book shows six triangles with two angles marked on each.)

Triangle	A	B	C	D	E	F
Angles marked	115°, 25°	50°, 105°	40°, 115°	25°, 50°	115°, 50°	40°, 25°

23.1.3 The triangles below are similar.

(Triangle MNO has ON = 6 cm, NM = 4 cm and OM = 10 cm. Triangle XYZ is the larger triangle, with ZX = 15 cm. O matches Z, N matches Y and M matches X; the angles at M and X are marked equal.)

(a) Calculate the length XY. [1]

(b) Calculate the length YZ. [1]

23.1.4 In the triangle calculate the lengths of sides p, q and r. [4]

(Diagram: a large triangle is cut by a line of length 3 cm drawn parallel to its left side p. The base is split into 3 cm (on the left, next to p) and 5 cm (on the right). The top side of the large triangle is 12 cm long; the cut splits it into q (on the left) and r (on the right).)

23.1.5 In the trapezium calculate the lengths of the sides e and f. [4]

(Diagram: a trapezium with a vertical left side made of 3 cm on top and 2 cm below. A line of length 6 cm runs parallel to the bottom side e, across from the point dividing the 3 cm and 2 cm parts. The sloping right side is 4 cm from the top down to that line, and f below it.)

23.1.6 The triangles PQR and LMN are similar. (Triangle PQR has base QR = 5 cm, perpendicular height 4 cm, angle Q = 70° and angle R = 50°. Triangle LMN has base MN = 8 cm, angle M = 70° and angle N = 50°.) Calculate:

(a) the area of ΔPQR [1]

(b) the scale factor of enlargement [1]

(c) the area of ΔLMN. [2]

23.1.7 The triangles ABC and XYZ below are similar. (Triangle ABC has a right angle at B, AB = 6 cm and BC = 8 cm. Triangle XYZ has a right angle at Y and hypotenuse XZ = 25 cm.)

(a) Using Pythagoras' theorem calculate the length of AC. [2]

(b) Calculate the scale factor of enlargement. [1]

(c) Calculate the area of ΔXYZ. [2]

23.1.8 The triangle ADE shown has an area of 12 cm^2. (Diagram: DE is parallel to BC, with D on AB and E on AC. The height of triangle ADE from A is 6 cm, and the height of the strip between DE and BC is 4 cm.)

(a) Calculate the area of ΔABC. [2]

(b) Calculate the length BC. [3]

23.1.9 The parallelograms below are similar. (The first parallelogram has sides 5 cm and 6 cm. The second has the side matching 5 cm equal to 8 cm, and the side matching 6 cm equal to x cm.) Calculate the length of the side marked x. [2]

23.1.10 The diagram below shows two rhombuses. (One rhombus has sides of 8 cm, the other has sides of 6 cm. No angles are given, and the two are drawn leaning at different angles.) Explain, giving reasons, whether the two rhombuses are definitely similar. [2]

23.1.11 The diagram shows a trapezium within a trapezium. Explain, giving reasons, whether the two trapezia are definitely similar. [2]

(Diagram: the outer trapezium has a parallel side of 10 cm and angles of 110° and 80° marked. The inner trapezium has a parallel side of 5 cm and also has angles of 110° and 80° marked. No other sides are given.)

Exercise 23.2 · Area of similar shapes

23.2.1 In the hexagons below, hexagon B is an enlargement of hexagon A by a scale factor of 2.5. If the area of A is 8 cm^2, calculate the area of B. [2]

23.2.2 P and Q are two regular pentagons. Q is an enlargement of P by a scale factor of 3. If the area of pentagon Q is 90 cm^2, calculate the area of P. [2]

23.2.3 The diagram below shows four triangles A, B, C and D. Each is an enlargement of the previous one by a scale factor of 1.5

(i) If the area of C is 202.5 cm^2, calculate the area of: triangle D [1]

(ii) triangle B [1]

(iii) triangle A. [1]

(b) If the triangles were to continue in this sequence, which letter triangle would be the first to have an area greater than 15 000 cm^2? [3]

23.2.4 A square is enlarged by increasing the length of its sides by 10%. If the length of its sides was originally 6 cm, calculate the area of the enlarged square. [2]

23.2.5 A square of side length 4 cm is enlarged by increasing the lengths of its sides by 25% and then increasing them by a further 50%. Calculate the area of the final square. [3]

23.2.6 An equilateral triangle has an area of 25 cm^2. If the lengths of its sides are reduced by 15%, calculate the area of the reduced triangle. [2]

Exercise 23.3 · Area and volume of similar shapes

23.3.1 The diagram shows a scale model of a garage. Its width is 5 cm, its length 10 cm and the height of its walls 6 cm. (The roof is a triangular prism on top of the walls; A is one rectangular roof section, 10 cm long, with slant length l.)

(i) If the width of the real garage is 4 m, calculate: the length of the real garage [1]

(ii) the real height of the garage wall. [1]

(b) If the apex of the roof of the real garage is 2 m above the top of the walls, use Pythagoras' theorem to find the real slant length l. [2]

(c) What is the area of the roof section A on the model? [3]

23.3.2 A cuboid has dimensions as shown in the diagram. (The cuboid is 10 cm long, 5 cm wide and 4 cm high.) If the cuboid is enlarged by a scale factor of 2.5, calculate:

(a) the total surface area of the original cuboid [2]

(b) the total surface area of the enlarged cuboid [2]

(c) the volume of the original cuboid [1]

(d) the volume of the enlarged cuboid. [2]

23.3.3 A cube has side length 3 cm.

(a) Calculate its total surface area. [1]

(b) If the cube is enlarged and has a total surface area of 486 cm^2, calculate the scale factor of enlargement. [2]

(c) Calculate the volume of the enlarged cube. [2]

23.3.4 Two cubes P and Q are of different sizes. If n is the ratio of their corresponding sides, express in terms of n:

(a) the ratio of their surface areas [1]

(b) the ratio of their volumes. [1]

23.3.5 The cuboids A and B shown below are similar. (Cuboid A has volume = 3024 cm^3 and a length of 15 cm. The matching length of cuboid B is 5 cm.) Calculate the volume of cuboid B. [3]

23.3.6 Two similar troughs X and Y are shown below. (The top length of trough X is 25 cm; the matching length of trough Y is 10 cm.) If the capacity of X is 10 litres, calculate the capacity of Y. [3]

Exercise 23.4 · More area and volume of similar shapes

23.4.1 The two cylinders L and M shown below are similar. (Cylinder L holds 16 litres; cylinder M holds 2 litres.) If the height of cylinder M is 10 cm, calculate the height of cylinder L. [3]

23.4.2 A square-based pyramid (below) is cut into two shapes by a cut running parallel to the base and made halfway up. (The two halves of the sloping edge are each marked x.)

(a) Calculate the ratio of the volume of the smaller pyramid to that of the original one. [2]

(b) Calculate the ratio of the volume of the small pyramid to that of the truncated base. [2]

23.4.3 The two cones A and B are similar. Cone B is an enlargement of A by a scale factor of 4. If the volume of cone B is 1024 cm^3, calculate the volume of cone A. [2]

23.4.4 (Diagram: a cylinder 20 cm long lies on its side. A shaded section 5 cm long is at one end, and the unshaded section is the remaining 15 cm. Both sections have the same radius.)

(a) Stating your reasons clearly, decide whether the two cylinders shown are similar or not. [2]

(b) What is the ratio of the curved surface area of the shaded cylinder to that of the unshaded cylinder? [2]

23.4.5 The diagram shows a triangle. (Diagram: triangle PST has a right angle at T. The base TS is split into TU = 8 cm, UV = 8 cm and VS = 8 cm. The lines UQ and VR are perpendicular to the base, with Q and R on PS, and VR = 4 cm.)

(a) Calculate the area of ΔRSV. [1]

(b) Calculate the area of ΔQSU. [2]

(c) Calculate the area of ΔPST. [2]

23.4.6 The area of an island on a map is 30 cm^2. The scale used on the map is 1 : 100 000.

(a) Calculate the area in square kilometres of the real island. [3]

(b) An airport on the island is on a rectangular piece of land measuring 3 km by 2 km. Calculate the area of the airport on the map in cm^2. [3]

23.4.7 The two packs of cheese X and Y are similar. The total surface area of pack Y is four times that of pack X. (Pack X measures 10 cm by 5 cm by 15 cm.) Calculate:

(a) the dimensions of pack Y [2]

(b) the mass of pack X if pack Y has a mass of 800 g. [2]

Exercise 23.5 · Congruent shapes

23.5.1 Look at the shapes on the grid below. Which shapes are congruent to shape A? [3]

(The book draws the shapes on a square grid. Their vertices, in order round each shape, with 1 unit = 1 grid square, are:)

Shape	Vertices
A	(2, 11), (6, 10), (3, 6), (1, 8)
P	(9, 12), (12, 11), (11, 7), (7, 10)
Q	(16, 11), (18, 9), (14, 6), (13, 10)
R	(7, 7), (8, 2), (5, 1), (3, 3)
S	(12, 6), (16, 2), (11, 1), (10, 4)

23.5.2 The two shapes below are congruent: (The first shape is an arrowhead-shaped quadrilateral with an angle of 40° and a reflex angle of 280° marked. The second shape is the same quadrilateral turned round, with an angle of 30° and an angle of x° marked.) Calculate the size of x. [2]

23.5.3 A quadrilateral is plotted on a pair of axes. The coordinates of its four vertices are (0, 1), (0, 5), (3, 4) and (3, 3). Another quadrilateral, congruent to the first, is also plotted on the same axes. Three of its vertices have coordinates (6, 5), (5, 2) and (4, 2). Calculate the coordinates of the fourth vertex. [3]

23.5.4 Triangle P is drawn on a graph. One side of another triangle, Q, is also shown. (Triangle P has vertices (1, 4), (5, 1) and (1, −1). The side of triangle Q runs from (7, 1) to (12, 1).) If triangles P and Q are congruent, give all the possible coordinates for the position of the missing vertex. [4]

Exercise 23.6 · Congruent triangles

23.6.1 Explain, justifying your answer, whether the two triangles below are definitely congruent. (Each triangle is right-angled, with angles of 30° and 60° marked. No lengths are given.) [2]

23.6.2 All three angles and sides of triangle X are given below: (Triangle X: the 8.0 cm side runs between the 120° and 25° angles, the 5.9 cm side between the 120° and 35° angles, and the 12.1 cm side between the 35° and 25° angles.) State which of the triangles A–E are definitely congruent to triangle X. Give reasons for your answer. [5]

Triangle	Information given in the diagram
A	side 5.9 cm, with angles 120° and 35° at its two ends
B	side 8 cm, with angles 35° and 25° at its two ends
C	sides 8.0 cm and 5.9 cm, with an angle of 25° at the far end of the 8.0 cm side (not between the two sides)
D	angles 120°, 35° and 25°, no lengths
E	sides 8 cm and 12.1 cm, with the angle of 25° between them

23.6.3 Regular hexagon ABCDEF is shown below: (The vertices are labelled in order round the hexagon.)

(a) Identify two triangles congruent to ΔACD. [2]

(b) Identify two triangles congruent to ΔDEF. [2]

23.6.4 Parallelogram PQRS is shown below: (P is bottom left, Q bottom right, R top right and S top left.) Explain, justifying your answer, whether ΔPQS is congruent to ΔQRS. [3]

Student assessment 1

SA1.1 The two triangles (below) are similar. (The first triangle is right-angled, with sides 3 cm and 4 cm either side of the right angle and hypotenuse p cm. The second triangle is right-angled, with the side matching 4 cm equal to 6 cm, the side matching 3 cm equal to x cm, and hypotenuse y cm.)

(a) Using Pythagoras' theorem, calculate the value of p. [2]

(b) Calculate the values of x and y. [2]

SA1.2 Cones M and N are similar. (Cone M has height h and cone N has height H.)

(a) Express the ratio of their surface areas in the form, area of M : area of N. [1]

(b) Express the ratio of their volumes in the form, volume of M : volume of N. [1]

SA1.3 Calculate the values of x, y and z in the triangle below. [5]

(Diagram: a large triangle has a right angle at its top vertex. Its left side is made of 5 cm and then 3 cm up to the top vertex. From the point between the 5 cm and 3 cm parts, a line of 4 cm is drawn at right angles to the left side, down to the base, parallel to the right side z cm. The base is split into x cm on the left and y cm on the right.)

SA1.4 The tins A and B are similar. The capacity of tin B is three times that of tin A. If the label on tin A has an area of 75 cm^2, calculate the area of the label on tin B. [3]

SA1.5 A cube of side 4 cm is enlarged by a scale factor of 2.5.

(a) Calculate the volume of the enlarged cube. [2]

(b) Calculate the surface area of the enlarged cube. [2]

SA1.6 The two troughs X and Y are similar. The scale factor of enlargement from Y to X is 4. If the capacity of trough X is 1200 cm^3, calculate the capacity of trough Y. (Trough X measures 12 cm, 8 cm and 20 cm.) [2]

SA1.7 The rectangular floor plan of a house measures 8 cm by 6 cm. If the scale of the plan is 1 : 50, calculate:

(a) the dimensions of the actual floor, [2]

(b) the area of the actual floor in m^2. [2]

SA1.8 The volume of the cylinder is 400 cm^3. (The cylinder is 10 cm high.) Calculate the volume of a similar cylinder formed by enlarging the one shown by a scale factor of 2. [2]

SA1.9 The diagram below, shows an equilateral triangle ABC. The midpoints L, M and N of each side are also joined. (L is the midpoint of AC, M of BC and N of AB.)

(a) Identify a trapezium congruent to trapezium BCLN. [1]

(b) Identify a triangle similar to ΔLMN. [1]

SA1.10 Decide whether each of the following statements is true or false.

(a) All circles are similar. [1]

(b) All squares are similar. [1]

(c) All rectangles are similar. [1]

(d) All equilateral triangles are congruent. [1]

Check yourself

1. Explain the difference between two shapes being similar and two shapes being congruent. [2]

2. A photo is enlarged so its lengths are 3 times as long. A student says the area is also 3 times as big. Explain the mistake and give the correct area factor. [2]

3. Two similar jugs hold 1 litre and 8 litres. The small jug is 12 cm tall. How tall is the big jug? [3]

4. Two triangles have two sides of 7 cm and 9 cm, and an angle of 40° that is not between them. Explain why they might not be congruent. [2]

5. Two similar statues have heights in the ratio 2 : 5. The small one needs 60 ml of paint. How much paint does the large one need? [3]

Chapter quiz

Q1 Two polygons are similar when they have...

A	B	C	D
Equal angles and sides in proportion	The same area	The same perimeter	The same number of sides

Q2 A 3, 4, 5 cm triangle is enlarged so the 3 cm side becomes 12 cm. How long is the side that was 5 cm?

A	B	C	D
14 cm	15 cm	17 cm	20 cm

Q3 The lengths of two similar shapes are in the ratio 1 : 3. What is the ratio of their areas?

A	B	C	D
1 : 3	1 : 6	1 : 9	1 : 27

Q4 Two similar solids have length scale factor 2. The small one has volume 15 cm^3. What is the big one's volume?

A	B	C	D
30 cm^3	45 cm^3	60 cm^3	120 cm^3

Q5 Two similar shapes have areas 25 cm^2 and 100 cm^2. What is the length scale factor?

A	B	C	D
2	4	5	75

Q6 Two similar jars hold 8 litres and 27 litres. What is the ratio of their heights?

A	B	C	D
2 : 3	4 : 9	8 : 27	1 : 3

Q7 The sides of a square are increased by 50%. By what percentage does its area increase?

A	B	C	D
50%	100%	125%	225%

Q8 Which pair of shapes is always similar?

A	B	C	D
Any two squares	Any two rectangles	Any two rhombuses	Any two isosceles triangles

Q9 Which of these is NOT a test for congruent triangles?

A	B	C	D
SSS	SAS	AAA	RHS

Q10 Two triangles are congruent. One has angles of 45° and 75°. What is the third angle of the other?

A	B	C	D
45°	60°	75°	90°

Q11 Two right-angled triangles both have a 13 cm hypotenuse and a 5 cm side. Which test proves them congruent?

A	B	C	D
RHS	SAS	AAA	ASA

Q12 A map has scale 1 : 1000. A field covers 5 cm² on the map. What is its real area?

A	B	C	D
50 m²	500 m²	5000 m²	0.5 m²

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-23-similarity-and-congruence/