C21

Straight line graphs

CHAPTER ASSIGNMENT · 387 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Use graph paper for every question that asks you to draw, plot or solve graphically. Gradient = $(y_2 - y_1) \div (x_2 - x_1)$. Exercises 21.13 to 21.15 are Extended content.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 21 (Straight-line graphs), for classroom use. Where the book draws a line on a grid, the line is described here by points on it; the number lines of Exercise 21.4 and the grids of Exercise 21.5 are in the book itself. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 21.1 · Coordinates

21.1.1 Draw a pair of axes with both x and y from −8 to +8. Mark each of the following points on your grid: (a) A = (5, 2) · (b) B = (7, 3) · (c) C = (2, 4) · (d) D = (−8, 5) · (e) E = (−5, −8) · (f) F = (3, −7) · (g) G = (7, −3) · (h) H = (6, −6) [8]

2 Draw a separate grid for each of Q.2–4 with x- and y-axes from −6 to +6. Plot and join the points in order to name each shape drawn.

21.1.2 A = (3, 2) · B = (3, −4) · C = (−2, −4) · D = (−2, 2) [1]

21.1.3 E = (1, 3) · F = (4, −5) · G = (−2, −5) [1]

21.1.4 H = (−6, 4) · I = (0, −4) · J = (4, −2) · K = (−2, 6) [1]

Exercise 21.2 · Shapes on a grid

6 Draw a pair of axes with both x and y from −10 to +10.

21.2.1 Plot the points P = (−6, 4), Q = (6, 4) and R = (8, −2). Plot point S such that PQRS when drawn is a parallelogram.

(a) Draw diagonals PR and QS. What are the coordinates of their point of intersection? [2]

(b) What is the area of PQRS? [2]

21.2.2 On the same axes, plot point M at (−8, 4) and point N at (4, 4).

(a) Join points MNRS. What shape is formed? [1]

(b) What is the area of MNRS? [2]

(c) Explain your answer to Q.2b. [2]

21.2.3 On the same axes, plot point J where point J has y-coordinate +10 and JRS, when joined, forms an isosceles triangle. What is the x-coordinate of all points on the line of symmetry of triangle JRS? [2]

Exercise 21.3 · Symmetric shapes

21.3.1 (a) On a grid with axes numbered from −10 to +10 draw a hexagon ABCDEF with centre (0, 0), points A (0, 8) and B(7, 4) and two lines of symmetry. [2]

(b) Write down the coordinates of points C, D, E and F. [4]

21.3.2 (a) On a similar grid to Q.1, draw an octagon PQRSTUVW which has point P(2, −8), point Q(−6, −8) and point R(−7, −5). PQ = RS = TU = VW and QR = ST = UV = WP. [2]

(b) List the coordinates of points S, T, U, V, and W. [5]

(c) What are the coordinates of the centre of rotational symmetry of the octagon? [1]

Exercise 21.4 · Reading scales

21.4.1 The four number lines for this exercise are printed in the book (pp. 251–252). Read the positions from the book's diagrams. The points A, B, C and D are not at whole number points on the number line. Point A is at 0.7. What are the positions of points B, C and D? (The number line runs from 0 to 6.) [3]

21.4.2 On this number line, point E is at 0.4. What are the positions of points F, G and H? (The number line runs from 0 to 2.) [3]

21.4.3 What are the positions of points I, J, K, L and M? (The number line runs from 4 to 7.) [5]

21.4.4 Point P is at position 0.4 and point W is at position 9.8. What are the positions of points Q, R, S, T, U, and V? (The number line runs from 0 to 10.) [6]

Exercise 21.5 · Reading coordinates

21.5.1 The two grids for this exercise are printed in the book (p. 252): the first has axes from −2 to 2, the second from −4 to 4. Give the coordinates of points A, B, C, D, E, F, G and H. [8]

Exercise 21.6 · The gradient of a straight line

21.6.1 For each of the following lines, select two points on the line and then calculate its gradient. (The book draws each line on a grid; here each line is described by points on it.) [12]

(a) The line through (−5, −4), (−1, 0) and (3, 4).

(b) The line through (0, −2), (1, 0) and (3, 4).

(c) The line through (−4, 5), (−2, 1) and (0, −3).

(d) The line through (−6, 2.5), (0, 1) and (4, 0).

(e) The horizontal line through (−4, −1) and (4, −1).

(f) The vertical line through (3, −4) and (3, 4).

21.6.2 From your answers to Question 1e, what conclusion can you make about the gradient of any horizontal line? [1]

21.6.3 From your answers to Question 1f, what conclusion can you make about the gradient of any vertical line? [1]

21.6.4 The graph in the book shows six straight lines labelled A–F. In words: A is a steep line rising to the right; B is a horizontal line; C is a steep line falling to the right, steeper than A; D falls gently to the right; E rises gently to the right; F is a vertical line. Six gradients are given below. Deduce which line has which gradient. [6]

Gradient = ½	Gradient is infinite	Gradient = 2	Gradient = −3	Gradient = 0	Gradient = −½

Exercise 21.7 · The equation of a straight line

21.7.1 In each of the following identify the coordinates of some of the points on the line and use these to find the equation of the straight line. (Each line is described by points on it.) [8]

(a) The horizontal line through (0, 7) and (8, 7).

(b) The horizontal line through (0, 2) and (8, 2).

(c) The vertical line through (7, 0) and (7, 8).

(d) The vertical line through (3, 0) and (3, 8).

(e) The line through (0, 0), (4, 4) and (8, 8).

(f) The line through (0, 0), (4, 2) and (8, 4).

(g) The line through (−4, 4), (−2, 2) and (0, 0).

(h) The line through (−4, 8), (−2, 4) and (0, 0).

Exercise 21.8 · Gradient and intercept

21.8.1 In each of the following, identify the coordinates of some of the points on the line and use these to find the equation of the straight line. (Each line is described by points on it.) [6]

(a) The line through (−3, −2), (0, 1) and (3, 4).

(b) The line through (−3, 0), (0, 3) and (2, 5).

(c) The line through (−2, −4), (0, −2) and (3, 1).

(d) The line through (−2, −2), (0, 2) and (3, 8).

(e) The line through (−4, 3), (0, 5) and (4, 7).

(f) The line through (−4, −3), (0, −1) and (4, 1).

21.8.2 In each of the following identify the coordinates of some of the points on the line and use these to find the equation of the straight line. (Each line is described by points on it.) [6]

(a) The line through (−4, 8), (0, 4) and (3, 1).

(b) The line through (−4, 2), (0, −2) and (2, −4).

(c) The line through (−4, 6), (0, −2) and (1, −4).

(d) The line through (−4, 5), (0, 3) and (2, 2).

(e) The line through (−4, 8), (0, 2) and (2, −1).

(f) The line through (−2, 9), (0, 1) and (1, −3).

21.8.3 (a) For each of the graphs in Questions 1 and 2 calculate the gradient of the straight line. [6]

(b) What do you notice about the gradient of each line and its equation? [1]

(c) What do you notice about the equation of the straight line and where the line intersects the y-axis? [1]

21.8.4 Copy the diagrams in Question 1. Draw two lines on each diagram parallel to the given line.

(a) Write the equation of these new lines in the form y = mx + c. [6]

(b) What do you notice about the equations of these new parallel lines? [1]

21.8.5 In Question 2, you have an equation for these lines in the form y = mx + c. Change the value of the intercept c and then draw the new line. What do you notice about this new line and the first line? [2]

Exercise 21.9 · y = mx + c and ax + by + c = 0

21.9.1 For Questions 1 to 7, calculate both the gradient and y-intercept in each case. A fraction such as (y + 2)/4 means the whole of y + 2 divided by 4. (a) y = 2x + 1 · (b) y = 3x + 5 · (c) y = x − 2 · (d) y = ½x + 4 · (e) y = −3x + 6 · (f) y = −⅔x + 1 · (g) y = −x · (h) y = −x − 2 · (i) y = −(2x − 2) [9]

21.9.2 (a) y − 3x = 1 · (b) y + ½x − 2 = 0 · (c) y + 3 = −2x · (d) y + 2x + 4 = 0 · (e) y − ¼x − 6 = 0 · (f) −3x + y = 2 · (g) 2 + y = x · (h) 8x − 6 + y = 0 · (i) −(3x + 1) + y = 0 [9]

21.9.3 (a) $2y = 4x - 6$ · (b) $2y = x + 8$ · (c) $\frac{1}{2}y = x - 2$ · (d) $\frac{1}{4}y = -2x + 3$ · (e) $3y - 6x = 0$ · (f) $\frac{1}{3}y + x = 1$ · (g) $6y - 6 = 12x$ · (h) $4y - 8 + 2x = 0$ · (i) $2y - (4x - 1) = 0$ [9]

21.9.4 (a) $2x - y = 4$ · (b) $x - y + 6 = 0$ · (c) $-2y = 6x + 2$ · (d) $12 - 3y = 3x$ · (e) $5x - \frac{1}{2}y = 1$ · (f) $-\frac{2}{3}y + 1 = 2x$ · (g) $9x - 2 = -y$ · (h) $-3x + 7 = -\frac{1}{2}y$ · (i) $-(4x - 3) = -2y$ [9]

21.9.5 (a) $(y + 2)/4 = \frac{1}{2}x$ · (b) $(y - 3)/x = 2$ · (c) $(y - x)/8 = 0$ · (d) $(2y - 3x)/2 = 6$ · (e) $(3y - 2)/x = -3$ · (f) $(\frac{1}{2}y - 1)/x = -2$ · (g) $(3x - y)/2 = 6$ · (h) $(6 - 2y)/3 = 2$ · (i) $-(x + 2y)/5x = 1$ [9]

21.9.6 (a) $(3x - y)/y = 2$ · (b) $(-x + 2y)/4 = y + 1$ · (c) $(y - x)/(x + y) = 2$ · (d) $1/y = 1/x$ · (e) $-(6x + y)/2 = y + 1$ · (f) $(2x - 3y + 4)/4 = 4$ [6]

21.9.7 (a) $(y + 1)/x + (3y - 2)/2x = -1$ · (b) $x/(y + 1) + 1/(2y + 2) = 3$ · (c) $-(-y + 3x)/-(6x - 2y) = 1$ · (d) $[-(x - 2y) - (-x - 2y)]/(4 + x - y) = -2$ [4]

21.9.8 Write each of the following equations in the form ax + by + c = 0 where a, b and c are integers. (a) y = ⅓x + 1 · (b) y = ⅖x − 2 · (c) 3y = (3/2)x − ½ · (d) −½y = x + ⅓ · (e) (y + 1)/2 = ¾x − 1 · (f) −(3/5)y − 4 = x [6]

21.9.9 Write each of the following equations in the form ax + by + c = 0 where a, b and c are integers. (a) (y − 1)/x − (y − 2)/3x = 1 · (b) (2y + x)/(x − y) = ½ · (c) 3/y = −2/5x · (d) −(2x + 3y)/3x = ⅔ · (e) 3(x − 2y)/x − 4 = 0 · (f) 2x/5y + x/−y = −1 [6]

Exercise 21.10 · Parallel lines and their equations

21.10.1 A straight line has equation 3y − 3x = 4. Write down the equation of another straight line parallel to it. [1]

21.10.2 A straight line has equation y = −x + 6. Which of the following lines is/are parallel to it? (a) 2(y + x) = −5 · (b) −3x − 3y + 7 = 0 · (c) 2y = −x + 12 · (d) y + x = 1/10 [4]

21.10.3 Find the equation of the line parallel to y = 4x − 1 that passes through (0, 0). [1]

21.10.4 Find the equations of lines parallel to y = −3x + 1 that pass through each of the following points: (a) (0, 4) · (b) (−2, 4) · (c) (−5/2, 4) [6]

21.10.5 Find the equations of lines parallel to $x - 2y = 6$ that pass through each of the following points: (a) $(-4, 1)$ · (b) $(\frac{1}{2}, 0)$ [4]

Exercise 21.11 · Drawing straight-line graphs

21.11.1 Plot the following straight lines. (a) $y = 2x + 3$ · (b) $y = x - 4$ · (c) $y = 3x - 2$ · (d) $y = -2x$ · (e) $y = -x - 1$ · (f) $-y = x + 1$ · (g) $-y = 3x - 3$ · (h) $2y = 4x - 2$ · (i) $y - 4 = 3x$ [9]

21.11.2 Plot the following straight lines: (a) $-2x + y = 4$ · (b) $-4x + 2y = 12$ · (c) $3y = 6x - 3$ · (d) $2x = x + 1$ · (e) $3y - 6x = 9$ · (f) $2y + x = 8$ · (g) $x + y + 2 = 0$ · (h) $3x + 2y - 4 = 0$ · (i) $4 = 4y - 2x$ [9]

21.11.3 Plot the following straight lines: (a) $(x + y)/2 = 1$ · (b) $x + y/2 = 1$ · (c) $x/3 + y/2 = 1$ · (d) $y + x/2 = 3$ · (e) $y/5 + x/3 = 0$ · (f) $-(2x + y)/4 = 1$ · (g) $(y - (x - y))/3x = -1$ · (h) $y/(2x + 3) - \frac{1}{2} = 0$ · (i) $-2(x + y) + 4 = -y$ [9]

Exercise 21.12 · Graphical solution of simultaneous equations

21.12.1 Solve the simultaneous equations below: (i) by graphical means, (ii) by algebraic means. (a) $x + y = 5$ and $x - y = 1$ · (b) $x + y = 7$ and $x - y = 3$ · (c) $2x + y = 5$ and $x - y = 1$ · (d) $2x + 2y = 6$ and $2x - y = 3$ · (e) $x + 3y = -1$ and $x - 2y = -6$ · (f) $x - y = 6$ and $x + y = 2$ [12]

21.12.2 (a) $3x - 2y = 13$ and $2x + y = 4$ · (b) $4x - 5y = 1$ and $2x + y = -3$ · (c) $x + 5 = y$ and $2x + 3y - 5 = 0$ · (d) $x = y$ and $x + y + 6 = 0$ · (e) $2x + y = 4$ and $4x + 2y = 8$ · (f) $y - 3x = 1$ and $y = 3x - 3$ [12]

Exercise 21.13 · Length and midpoint of a line segment (Extended)

21.13.1 (i) Plot each of the following pairs of points. (ii) Calculate the distance between each pair of points. (iii) Find the coordinates of the midpoint of the line segment joining the two points.

a	b	c	d	e	f
(5, 6) (1, 2)	(6, 4) (3, 1)	(1, 4) (5, 8)	(0, 0) (4, 8)	(2, 1) (4, 7)	(0, 7) (−3, 1)
g	h	i	j	k	l
(−3, −3) (−1, 5)	(4, 2) (−4, −2)	(−3, 5) (4, 5)	(2, 0) (2, 6)	(−4, 3) (4, 5)	(3, 6) (−3, −3)

21.13.2 Without plotting the points: (i) calculate the distance between each of the following pairs of points (ii) find the coordinates of the midpoint of the line segment joining the two points.

a	b	c	d	e	f
(1, 4) (4, 1)	(3, 6) (7, 2)	(2, 6) (6, −2)	(1, 2) (9, −2)	(0, 3) (−3, 6)	(−3, −5) (−5, −1)
g	h	i	j	k	l
(−2, 6) (2, 0)	(2, −3) (8, 1)	(6, 1) (−6, 4)	(−2, 2) (4, −4)	(−5, −3) (6, −3)	(3, 6) (5, −2)

Exercise 21.14 · The equation of a line through two points (Extended)

21.14.1 Find the equation of the straight line which passes through each of the following pairs of points. (a) (1, 1) (4, 7) · (b) (1, 4) (3, 10) · (c) (1, 5) (2, 7) · (d) (0, −4) (3, −1) · (e) (1, 6) (2, 10) · (f) (0, 4) (1, 3) · (g) (3, −4) (10, −18) · (h) (0, −1) (1, −4) · (i) (0, 0) (10, 5) [18]

21.14.2 (a) (−5, 3) (2, 4) · (b) (−3, −2) (4, 4) · (c) (−7, −3) (−1, 6) · (d) (2, 5) (1, −4) · (e) (−3, 4) (5, 0) · (f) (6, 4) (−7, 7) · (g) (−5, 2) (6, 2) · (h) (1, −3) (−2, 6) · (i) (6, −4) (6, 6) [18]

Exercise 21.15 · Perpendicular lines (Extended)

21.15.1 Calculate: (i) the gradient of the line joining the following pairs of points (ii) the gradient of a line perpendicular to this line (iii) the equation of the perpendicular line if it passes through the second point each time.

(a) (1, 4) (4, 1) · (b) (3, 6) (7, 2) · (c) (2, 6) (6, −2) · (d) (1, 2) (9, −2) · (e) (0, 3) (−3, 6) · (f) (−3, −5) (−5, −1)

(g) (−2, 6) (2, 0) · (h) (2, −3) (8, 1) · (i) (6, 1) (−6, 4) · (j) (−2, 2) (4, −4) · (k) (−5, −3) (6, −3) · (l) (3, 6) (5, −2)

21.15.2 The diagram in the book shows a square ABCD. The coordinates of A and B are given: A(4, 3) and B(10, 5). Going round the square, the corners are A (bottom left), B (bottom right), C (top right) and D (top left). Calculate:

(a) the gradient of the line AB [1]

(b) the equation of the line passing through A and B [2]

(c) the gradient of the line AD [1]

(d) the equation of the line passing through A and D [2]

(e) the equation of the line passing through B and C [2]

(f) the coordinates of C [2]

(g) the coordinates of D [2]

(h) the equation of the line passing through C and D [2]

(i) the length of the sides of the square to 1 d.p. [2]

(j) the coordinates of the midpoint of the line segment AC. [2]

21.15.3 The diagram in the book shows a right-angled isosceles triangle ABC, where AB = AC. The coordinates of A and B are given: A(−1, −2) and B(9, 2). C is above and to the left of A. Calculate:

(a) the equation of the line passing through the points A and B [2]

(b) the equation of the line passing through A and C [2]

(c) the length of the line segment BC to 1 d.p. [3]

(d) the coordinates of the midpoints of all three sides of the triangle. [3]

Student assessment 1

SA1.1 For each of the following lines, select two points on the line and then calculate its gradient. (Each line is described by points on it.) [4]

(a) The line through (−4, −3), (0, 1) and (4, 5).

(b) The line through (−2, 9), (1, 0) and (2, −3).

SA1.2 Find the equation of the straight line for each of the following: (Each line is described by points on it.) [4]

(a) The line through (−3, −2), (−2, 0) and (2, 8).

(b) The line through (−2, −1), (0, 4) and (2, 9).

SA1.3 Calculate the gradient and y-intercept for each of the following linear equations: (a) $y = -3x + 4$ · (b) $\frac{1}{3}y - x = 2$ · (c) $2x + 4y - 6 = 0$ **[6]**

SA1.4 Write down the equation of the line parallel to the line $y = -\frac{2}{3}x + 4$ which passes through the point (6, 2). **[2]**

SA1.5 Plot the following graphs on the same pair of axes, labelling each clearly. (a) $x = -2$ · (b) $y = 3$ · (c) $y = 2x$ · (d) $y = -x/2$ **[4]**

SA1.6 Solve the following pairs of simultaneous equations graphically: (a) $x + y = 4$ and $x - y = 0$ · (b) $3x + y = 2$ and $x - y = 2$ · (c) $y + 4x + 4 = 0$ and $x + y = 2$ · (d) $x - y = -2$ and $3x + 2y + 6 = 0$ **[8]**

SA1.7 The coordinates of the end points of two line segments are given below. For each line segment calculate: (i) the length (ii) the midpoint. (a) (−6, −1) (6, 4) · (b) (1, 2) (7, 10) **[4]**

SA1.8 Find the equation of the straight line which passes through each of the following pairs of points: (i) in the form $y = mx + c$ (ii) in the form $ax + by + c = 0$. (a) (1, −1) (4, 8) · (b) (0, 7) (3, 1) **[8]**

SA1.9 A line L_1 passes through the points $(-2, 5)$ and $(5, 3)$.

(a) Write down the equation of the line L_1. [2]

(b) Another line L_2 is perpendicular to L_1 and also passes through the point $(-2, 5)$. Write down the equation of the line L_2. [3]

SA1.10 The diagram in the book shows a rhombus ABCD. The coordinates of A, B and D are given: A(4, 3), B(9, 3) and D(0, 6). C is the fourth corner.

(a) Calculate: (i) the coordinate of the point C · (ii) the equation of the line passing through A and C · (iii) the equation of the line passing through B and D. [6]

(b) Are the diagonals of the rhombus perpendicular to each other? Justify your answer. [2]

Check yourself

1. A line passes through (−2, 1) and (4, 4). Find its gradient, its equation in the form y = mx + c, and the point where it crosses the x-axis. [4]

2. Show that the points (1, 2), (3, 6) and (6, 12) all lie on the same straight line. [3]

3. P is (−3, 7) and Q is (5, 1). Find the length of PQ and the midpoint of PQ. [4]

4. Line L has equation 2x + 3y = 12. Find the equation of the line perpendicular to L that passes through (2, −1). [4]

5. Explain why the simultaneous equations y = 2x + 1 and 4x − 2y = 6 have no solution. Use the gradients in your answer. [3]

Chapter quiz

Q1 What is the gradient of the line through (1, 2) and (3, 8)?

A	B	C	D
2	6	3	1/3

Q2 In $y = 5x - 7$, what is the y-intercept?

A	B	C	D
5	−7	7	−5

Q3 What is the gradient of the horizontal line $y = 4$?

A	B	C	D
4	1	Infinite (undefined)	0

Q4 Which line is parallel to $y = 3x + 1$?

A	B	C	D
$y = -3x + 1$	$y = 3x - 5$	$y = \frac{1}{3}x + 1$	$y = x + 3$

Q5 What is the gradient of the line $2y = 6x - 4$?

A	B	C	D
6	3	−4	−2

Q6 A line has gradient 2. What is the gradient of a line perpendicular to it?

A	B	C	D
−2	½	−½	2

Q7 What is the midpoint of the line segment from (2, 3) to (8, 7)?

A	B	C	D
(6, 4)	(10, 10)	(5, 5)	(3, 2)

Q8 How long is the line segment from (0, 0) to (6, 8)?

A	B	C	D
14	10	48	100

Q9 The lines x + y = 6 and x − y = 2 are drawn. Where do they cross?

A	B	C	D
(2, 4)	(3, 3)	(6, 2)	(4, 2)

Q10 Which is y = ½x + 3 written in the form ax + by + c = 0?

A	B	C	D
x + 2y + 6 = 0	x − 2y + 6 = 0	2x − y + 3 = 0	x − 2y − 3 = 0

Q11 Which line has gradient −3 and passes through (0, 1)?

A	B	C	D
y = x − 3	y = −3x + 1	y = 3x − 1	y = −3x − 1

Q12 Which point lies on the line y = 2x − 1?

A	B	C	D
(3, 2)	(1, 2)	(2, 3)	(0, 1)

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-21-straight-line-graphs/