C11

Algebraic representation and manipulation

CHAPTER ASSIGNMENT · 57 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Work without a calculator unless a question says otherwise. A fraction bar in the book is written here with / and brackets, so (a + b)/c means a + b over c. The book prints the letter to make the subject in red or green; here it is given after "subject:" in bold.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 11 (Algebraic representation and manipulation), for classroom use. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 11.1 · Expanding a bracket

11.1.1 Expand the following:

(a) $4(x - 3)$	(b) $5(2p - 4)$	(c) $-6(7x - 4y)$
(d) $3(2a - 3b - 4c)$	(e) $-7(2m - 3n)$	(f) $-2(8x - 3y)$

11.1.2 Expand the following:

(a) $3x(x - 3y)$	(b) $a(a + b + c)$	(c) $4m(2m - n)$
(d) $-5a(3a - 4b)$	(e) $-4x(-x + y)$	(f) $-8p(-3p + q)$

11.1.3 Expand the following:

(a) $-(2x^2 - 3y^2)$	(b) $-(-a + b)$	(c) $-(-7p + 2q)$
(d) $(1/2)(6x - 8y + 4z)$	(e) $(3/4)(4x - 2y)$	(f) $(1/5)x(10x - 15y)$

11.1.4 Expand the following:

(a) $3r(4r^2 - 5s + 2t)$	(b) $a^2(a + b + c)$	(c) $3a^2(2a - 3b)$
(d) $pq(p + q - pq)$	(e) $m^2(m - n + nm)$	(f) $a^3(a^3 + a^2b)$

Exercise 11.2 · Expanding and simplifying

11.2.1 Expand and simplify the following:

(a) $3a - 2(2a + 4)$	(b) $8x - 4(x + 5)$	(c) $3(p - 4) - 4$
(d) $7(3m - 2n) + 8n$	(e) $6x - 3(2x - 1)$	(f) $5p - 3p(p + 2)$

11.2.2 Expand and simplify the following:

(a) $7m(m + 4) + m^2 + 2$	(b) $3(x - 4) + 2(4 - x)$	(c) $6(p + 3) - 4(p - 1)$
(d) $5(m - 8) - 4(m - 7)$	(e) $3a(a + 2) - 2(a^2 - 1)$	(f) $7a(b - 2c) - c(2a - 3)$

(a) $7m(m + 4) + m^2 + 2$	(b) $3(x - 4) + 2(4 - x)$	(c) $6(p + 3) - 4(p - 1)$

11.2.3 Expand and simplify the following:

(a) $(1/2)(6x + 4) + (1/3)(3x + 6)$	(b) $(1/4)(2x + 6y) + (3/4)(6x - 4y)$	(c) $(1/8)(6x - 12y) + (1/2)(3x - 2y)$
(d) $(1/5)(15x + 10y) + (3/10)(5x - 5y)$	(e) $(2/3)(6x - 9y) + (1/3)(9x + 6y)$	(f) $(x/7)(14x - 21y) - (x/2)(4x - 6y)$

Exercise 11.3 · Expanding a pair of brackets

11.3.1 Expand the following and simplify your answer:

(a) $(x + 2)(x + 3)$	(b) $(x + 3)(x + 4)$	(c) $(x + 5)(x + 2)$
(d) $(x + 6)(x + 1)$	(e) $(x - 2)(x + 3)$	(f) $(x + 8)(x - 3)$

11.3.2 Expand the following and simplify your answer:

(a) $(x - 4)(x + 6)$	(b) $(x - 7)(x + 4)$	(c) $(x + 5)(x - 7)$
(d) $(x + 3)(x - 5)$	(e) $(x + 1)(x - 3)$	(f) $(x - 7)(x + 9)$

11.3.3 Expand the following and simplify your answer:

(a) $(x - 2)(x - 3)$	(b) $(x - 5)(x - 2)$	(c) $(x - 4)(x - 8)$
(d) $(x + 3)(x + 3)$	(e) $(x - 3)(x - 3)$	(f) $(x - 7)(x - 5)$

11.3.4 Expand the following and simplify your answer:

(a) $(x + 3)(x - 3)$	(b) $(x + 7)(x - 7)$	(c) $(x - 8)(x + 8)$
(d) $(x + y)(x - y)$	(e) $(a + b)(a - b)$	(f) $(p - q)(p + q)$

Exercise 11.4 · Simple factorising

11.4.1 Factorise the following:

(a) 4x – 6	(b) 18 – 12p	(c) 6y – 3
(d) 4a + 6b	(e) 3p – 3q	(f) 8m + 12n + 16r

11.4.2 Factorise the following:

(a) 3ab + 4ac – 5ad	(b) 8pq + 6pr – 4ps	(c) a^2 – ab
(d) $4x^2$ – 6xy	(e) abc + abd + fab	(f) $3m^2$ + 9m

11.4.3 Factorise the following:

(a) 3pqr – 9pqs	(b) $5m^2$ – 10mn	(c) $8x^2y$ – $4xy^2$
(d) $2a^2b^2$ – $3b^2c^2$	(e) 12p – 36	(f) 42x – 54

11.4.4 Factorise the following:

(a) 18 + 12y	(b) 14a – 21b	(c) 11x + 11xy
(d) 4s – 16t + 20r	(e) 5pq – 10qr + 15qs	(f) 4xy + $8y^2$

11.4.5 Factorise the following:

(a) m^2 + mn	(b) $3p^2$ – 6pq	(c) pqr + qrs
(d) ab + a^2b + ab^2	(e) $3p^3$ – $4p^4$	(f) $7b^3c$ + b^2c^2

11.4.6 Factorise the following:

(a) m^3 – m^2n + mn^2	(b) $4r^3$ – $6r^2$ + $8r^2s$
(c) $56x^2y$ – $28xy^2$	(d) $72m^2n$ + $36mn^2$ – $18m^2n^2$

Exercise 11.5 · Substitution

11.5.1 Evaluate the following expressions if p = 4, q = −2, r = 3 and s = −5:

(a) 2p + 4q	(b) 5r − 3s	(c) 3q − 4s
(d) 6p − 8q + 4s	(e) 3r − 3p + 5q	(f) −p − q + r + s

11.5.2 Using p = 4, q = −2, r = 3 and s = −5, evaluate:

(a) 2p − 3q − 4r + s	(b) 3s − 4p + r + q	(c) $p^2 + q^2$
(d) $r^2 - s^2$	(e) p(q − r + s)	(f) r(2p − 3q)

11.5.3 Using p = 4, q = −2, r = 3 and s = −5, evaluate:

(a) 2s(3p − 2q)	(b) pq + rs	(c) 2pr − 3rq
(d) $q^3 - r^2$	(e) $s^3 - p^3$	(f) $r^4 - q^5$

11.5.4 Using p = 4, q = −2, r = 3 and s = −5, evaluate:

(a) −2pqr	(b) −2p(q + r)	(c) −2rq + r
(d) (p + q)(r − s)	(e) (p + s)(r − q)	(f) (r + q)(p − s)

11.5.5 Using p = 4, q = −2, r = 3 and s = −5, evaluate:

(a) (2p + 3q)(p − q)	(b) (q + r)(q − r)	(c) $q^2 - r^2$
(d) $p^2 - r^2$	(e) (p + r)(p − r)	(f) $(-s + p)q^2$

Exercise 11.6 · Rearrangement of formulae

11.6.1 Make the given letter the subject of the formula:

(a) m + n = r; subject: **n**	(b) m + n = p; subject: **m**	(c) 2m + n = 3p; subject: **n**
(d) 3x = 2p + q; subject: **q**	(e) ab = cd; subject: **a**	(f) ab = cd; subject: **d**

11.6.2 Make the given letter the subject of the formula:

(a) 3xy = 4m; subject: x	**(b) 7pq = 5r; subject: r**	**(c) 3x = c; subject: x**
(d) 3x + 7 = y; subject: **x**	(e) 5y − 9 = 3r; subject: **y**	(f) 5y − 9 = 3x; subject: **x**

11.6.3 Make the given letter the subject of the formula:

(a) 6b = 2a − 5; subject: b	**(b) 6b = 2a − 5; subject: a**	**(c) 3x − 7y = 4z; subject: z**
(d) 3x − 7y = 4z; subject: **x**	(e) 3x − 7y = 4z; subject: **y**	(f) 2pr − q = 8; subject: **p**

11.6.4 Make the given letter the subject of the formula:

(a) p/4 = r; subject: p	**(b) 4/p = 3r; subject: p**	**(c) (1/5)n = 2p; subject: p**
(d) (1/5)n = 2p; subject: **n**	(e) p(q + r) = 2t; subject: **p**	(f) p(q + r) = 2t; subject: **q**

11.6.5 Make the given letter the subject of the formula 3m − n = rt(p + q):

(a) subject: r	**(b) subject: t**	**(c) subject: m**
(d) subject: **n**	(e) subject: **p**	(f) subject: **q**

11.6.6 Make the given letter the subject of the formula:

(a) ab/c = de; subject: d	**(b) ab/c = de; subject: a**	**(c) ab/c = de; subject: c**
(d) (a + b)/c = d; subject: **a**	(e) a/c + b = d; subject: **b**	(f) a/c + b = d; subject: **c**

Exercise 11.7 · Further expansion (Extended)

11.7.1 Expand the following brackets, giving your answer in its simplest form:

(a) (y + 2)(2y + 3)	**(b) (y + 7)(3y + 4)**	**(c) (2y + 1)(y + 8)**
(d) (2y + 1)(2y + 2)	(e) (3y + 4)(2y + 5)	(f) (6y + 3)(3y + 1)

11.7.2 Expand the following brackets, giving your answer in its simplest form:

(a) (2p – 3)(p + 8)	**(b) (4p – 5)(p + 7)**	**(c) (3p – 4)(2p + 3)**
(d) (4p – 5)(3p + 7)	(e) (6p + 2)(3p – 1)	(f) (7p – 3)(4p + 8)

11.7.3 Expand the following brackets, giving your answer in its simplest form:

(a) (2x – 1)(2x – 1)	**(b) $(3x + 1)^2$**	**(c) $(4x – 2)^2$**
(d) $(5x – 4)^2$	(e) $(2x + 6)^2$	(f) (2x + 3)(2x – 3)

11.7.4 Expand the following brackets, giving your answer in its simplest form:

(a) (3 + 2x)(3 – 2x)	**(b) (4x – 3)(4x + 3)**	**(c) (3 + 4x)(3 – 4x)**
(d) (7 – 5y)(7 + 5y)	(e) (3 + 2y)(4y – 6)	(f) $(7 – 5y)^2$

11.7.5 Expand the following brackets, giving your answer in its simplest form:

(a) (x + 3)(3x + 1)(x + 2)	**(b) (2x + 4)(2x + 1)(x – 2)**	**(c) (–x + 1)(3x – 1)(4x + 3)**
(d) (–2x – 3)(–x + 1)(–x + 5)	(e) $(2x^2 – 3x + 1)(–x + 4)$	(f) $(4x – 1)(–3x^2 – 3x – 2)$

Exercise 11.8 · Factorisation by grouping (Extended)

11.8.1 Factorise the following by grouping:

(a) ax + bx + ay + by	**(b) ax + bx – ay – by**	**(c) 3m + 3n + mx + nx**
(d) 4m + mx + 4n + nx	(e) 3m + mx – 3n – nx	(f) 6x + xy + 6z + zy

11.8.2 Factorise the following by grouping:

(a) pr – ps + qr – qs	**(b) pq – 4p + 3q – 12**	**(c) pq + 3q – 4p – 12**
(d) $rs + rt + 2ts + 2t^2$	(e) $rs – 2ts + rt – 2t^2$	(f) $ab – 4cb + ac – 4c^2$

11.8.3 Factorise the following by grouping:

(a) $xy + 4y + x^2 + 4x$	(b) $x^2 - xy - 2x + 2y$	(c) $ab + 3a - 7b - 21$
(d) $ab - b - a + 1$	(e) $pq - 4p - 4q + 16$	(f) $mn - 5m - 5n + 25$

11.8.4 Factorise the following by grouping:

(a) $mn - 2m - 3n + 6$	(b) $mn - 2mr - 3rn + 6r^2$	(c) $pr - 4p - 4qr + 16q$
(d) $ab - a - bc + c$	(e) $x^2 - 2xz - 2xy + 4yz$	(f) $2a^2 + 2ab + b^2 + ab$

Exercise 11.9 · Difference of two squares (Extended)

11.9.1 Factorise the following:

(a) $a^2 - b^2$	(b) $m^2 - n^2$	(c) $x^2 - 25$
(d) $m^2 - 49$	(e) $81 - x^2$	(f) $100 - y^2$

11.9.2 Factorise the following:

(a) $144 - y^2$	(b) $q^2 - 169$	(c) $m^2 - 1$
(d) $1 - t^2$	(e) $4x^2 - y^2$	(f) $25p^2 - 64q^2$

11.9.3 Factorise the following:

(a) $9x^2 - 4y^2$	(b) $16p^2 - 36q^2$	(c) $64x^2 - y^2$
(d) $x^2 - 100y^2$	(e) $(qr)^2 - 4p^2$	(f) $(ab)^2 - (cd)^2$

11.9.4 Factorise the following:

(a) $m^2n^2 - 9y^2$	(b) $(1/4)x^2 - (1/9)y^2$	(c) $(2x)^2 - (3y)^4$
(d) $p^4 - q^4$	(e) $4m^4 - 36y^4$	(f) $16x^4 - 81y^4$

Exercise 11.10 · Evaluation (Extended)

11.10.1 By factorising, evaluate the following:

(a) $8^2 - 2^2$	(b) $16^2 - 4^2$	(c) $49^2 - 1$
(d) $17^2 - 3^2$	(e) $88^2 - 12^2$	(f) $96^2 - 4^2$

11.10.2 By factorising, evaluate the following:

(a) $45^2 - 25$	(b) $99^2 - 1$	(c) $27^2 - 23^2$
(d) $66^2 - 34^2$	(e) $999^2 - 1$	(f) $225 - 8^2$

11.10.3 By factorising, evaluate the following:

(a) $8.4^2 - 1.6^2$	(b) $9.3^2 - 0.7^2$	(c) $42.8^2 - 7.2^2$
(d) $(8\frac{1}{2})^2 - (1\frac{1}{2})^2$	(e) $(7\frac{3}{4})^2 - (2\frac{1}{4})^2$	(f) $5.25^2 - 4.75^2$

11.10.4 By factorising, evaluate the following:

(a) $8.62^2 - 1.38^2$	(b) $0.9^2 - 0.1^2$	(c) $3^4 - 2^4$
(d) $2^4 - 1$	(e) $1111^2 - 111^2$	(f) $2^8 - 25$

Exercise 11.11 · Factorising quadratic expressions (Extended)

11.11.1 Factorise the following quadratic expressions:

(a) $x^2 + 7x + 12$	(b) $x^2 + 8x + 12$	(c) $x^2 + 13x + 12$
(d) $x^2 - 7x + 12$	(e) $x^2 - 8x + 12$	(f) $x^2 - 13x + 12$

11.11.2 Factorise the following quadratic expressions:

(a) $x^2 + 6x + 5$	(b) $x^2 + 6x + 8$	(c) $x^2 + 6x + 9$

(a) $x^2 + 6x + 5$	(b) $x^2 + 6x + 8$	(c) $x^2 + 6x + 9$
(d) $x^2 + 10x + 25$	(e) $x^2 + 22x + 121$	(f) $x^2 - 13x + 42$

11.11.3 Factorise the following quadratic expressions:

(a) $x^2 + 14x + 24$	(b) $x^2 + 11x + 24$	(c) $x^2 - 10x + 24$
(d) $x^2 + 15x + 36$	(e) $x^2 + 20x + 36$	(f) $x^2 - 12x + 36$

11.11.4 Factorise the following quadratic expressions:

(a) $x^2 + 2x - 15$	(b) $x^2 - 2x - 15$	(c) $x^2 + x - 12$
(d) $x^2 - x - 12$	(e) $x^2 + 4x - 12$	(f) $x^2 - 15x + 36$

11.11.5 Factorise the following quadratic expressions:

(a) $x^2 - 2x - 8$	(b) $x^2 - x - 20$	(c) $x^2 + x - 30$
(d) $x^2 - x - 42$	(e) $x^2 - 2x - 63$	(f) $x^2 + 3x - 54$

11.11.6 Factorise the following quadratic expressions:

(a) $2x^2 + 3x + 1$	(b) $2x^2 + 7x + 6$	(c) $2x^2 + x - 6$
(d) $2x^2 - 7x + 6$	(e) $3x^2 + 8x + 4$	(f) $3x^2 + 11x - 4$
(g) $4x^2 + 12x + 9$	(h) $9x^2 - 6x + 1$	(i) $6x^2 - x - 1$

Exercise 11.12 · Rearrangement of complex formulae (Extended)

11.12.1 In the formulae below, make x the subject:

(a) $P = 2mx$	(b) $T = 3x^2$	(c) $mx^2 = y^2$
(d) $x^2 + y^2 = p^2 - q^2$	(e) $m^2 + x^2 = y^2 - n^2$	(f) $p^2 - q^2 = 4x^2 - y^2$

(a) $P = 2mx$	(b) $T = 3x^2$	(c) $mx^2 = y^2$

11.12.2 In the formulae below, make x the subject:

(a) $P/Q = rx$	(b) $P/Q = rx^2$	(c) $P/Q = x^2/r$
(d) $m/n = 1/x^2$	(e) $r/(st) = w/x^2$	(f) $(p + q)/r = w/x^2$

11.12.3 In the formulae below, make x the subject:

(a) $\sqrt{x} = rp$	(b) $mn/p = \sqrt{x}$	(c) $g = \sqrt{(k/x)}$
(d) $r = 2\pi\sqrt{(x/g)}$	(e) $p^2 = 4m^2r/x$	(f) $p = 2m\sqrt{(r/x)}$

Exercise 11.13 · More complex formulae (Extended)

11.13.1 Make the given letter the subject of the formula:

(a) $v = u + at$; subject: **t**	(b) $v^2 = u^2 + 2as$; subject: **u**	(c) $v^2 = u^2 + 2as$; subject: **s**
(d) $s = ut + (1/2)at^2$; subject: **u**	(e) $s = ut + (1/2)at^2$; subject: **a**	(f) $s = ut + (1/2)at^2$; subject: **t**

11.13.2 Make the given letter the subject of the formula:

(a) $A = \pi r\sqrt{(s^2 + t^2)}$; subject: **r**	(b) $A = \pi r\sqrt{(h^2 + r^2)}$; subject: **h**	(c) $1/f = 1/u + 1/v$; subject: **u**
(d) $1/f = 1/u + 1/v$; subject: **v**	(e) $t = 2\pi\sqrt{(l/g)}$; subject: **l**	(f) $t = 2\pi\sqrt{(l/g)}$; subject: **g**

11.13.3 Make the given letter the subject of the formula:

(a) $xt/7 = (p + 2)/(3x)$; subject: **x**	(b) $\sqrt{(a + 2)} = (b - 3)/\sqrt{(a - 2)}$; subject: **a**

Exercise 11.14 · Using formulae (Extended)

11.14.1 The volume of a cylinder is given by the formula $V = \pi r^2 h$, where h is the height of the cylinder and r is the radius. (Calculator allowed.)

(a) Find the volume of a cylindrical post of length 7.5 m and a diameter of 30 cm. [2]

(b) Make r the subject of the formula. [1]

(c) A cylinder of height 75 cm has a volume of 6000 cm^3, find its radius correct to 3 s.f. [2]

11.14.2 The formula C = (5/9)(F − 32) can be used to convert temperatures in degrees Fahrenheit (°F) into degrees Celsius (°C). (Calculator allowed.)

(a) What temperature in °C is equivalent to 150 °F? [1]

(b) What temperature in °C is equivalent to 12 °F? [1]

(c) Make F the subject of the formula. [1]

(d) Use your rearranged formula to find what temperature in °F is equivalent to 160 °C. [1]

11.14.3 The height of Mount Kilamanjaro is given as 5900 m. The formula for the time taken, T hours, to climb to a height H metres is T = H/1200 + k, where k is a constant. (Calculator allowed.)

(a) Calculate the time taken, to the nearest hour, to climb to the top of the mountain if k = 9.8. [1]

(b) Make H the subject of the formula. [1]

(c) How far up the mountain, to the nearest 100 m, could you expect to be after 14 hours? [2]

11.14.4 The formula for the volume V of a sphere is given as $V = (4/3)\pi r^3$. (Calculator allowed.)

(a) Find V if r = 5 cm. [1]

(b) Make r the subject of the formula. [1]

(c) Find the radius of a sphere of volume 2500 m^3. [2]

11.14.5 The cost \$x of printing n newspapers is given by the formula x = 1.50 + 0.05n.

(a) Calculate the cost of printing 5000 newspapers. [1]

(b) Make n the subject of the formula. [1]

(c) How many newspapers can be printed for \$25? [1]

Exercise 11.15 · Simplifying algebraic fractions (Extended)

11.15.1 Simplify the following algebraic fractions:

(a) $x/y \times p/q$	(b) $x/y \times q/x$	(c) $p/q \times q/r$
(d) $ab/c \times d/(ab)$	(e) $ab/c \times d/(ac)$	(f) $p^2/q^2 \times q^2/p$

11.15.2 Simplify the following algebraic fractions:

(a) m^3/m	(b) r^7/r^2	(c) x^9/x^3
(d) $x^2y^4/(xy^2)$	(e) $a^2b^3c^4/(ab^2c)$	(f) $pq^2r^4/(p^2q^3r)$

11.15.3 Simplify the following algebraic fractions:

(a) $4ax/(2ay)$	(b) $12pq^2/(3p)$	(c) $15mn^2/(3mn)$
(d) $24x^5y^3/(8x^2y^2)$	(e) $36p^2qr/(12pqr)$	(f) $16m^2n/(24m^3n^2)$

11.15.4 Simplify the following algebraic fractions:

(a) $2/b \times a/3$	(b) $4/x \times y/2$	(c) $8/x \times x/4$
(d) $9y/2 \times 2x/3$	(e) $12x/7 \times 7/(4x)$	(f) $4x^3/(3y) \times 9y^2/(2x^2)$

11.15.5 Simplify the following algebraic fractions:

(a) $2ax/(3bx) \times 4by/a$	(b) $3p^2/(2q) \times 5q/(3p)$
(c) $p^2q/(rs) \times pr/q$	(d) $a^2b/(fc^2) \times cd/(bd) \times ef^2/(ca^2)$

11.15.6 Simplify the following algebraic fractions:

(a) $8x^2/3 \div 2x/5$	(b) $3b^3/2 \div 4b^2/3$

Exercise 11.16 · Adding and subtracting fractions (Extended)

11.16.1 Simplify the following fractions:

(a) 1/7 + 3/7	(b) a/7 + b/7	(c) 5/13 + 6/13
(d) c/13 + d/13	(e) x/3 + y/3 + z/3	(f) $p^2/5 + q^2/5$

11.16.2 Simplify the following fractions:

(a) 5/11 − 2/11	(b) c/11 − d/11	(c) 6/a − 2/a
(d) 2a/3 − 5b/3	(e) 2x/7 − 3y/7	(f) 3/(4x) − 5/(4x)

11.16.3 Simplify the following fractions:

(a) 5/6 − 1/3	(b) 5/(2a) − 1/a	(c) 2/(3c) + 1/c
(d) 2/x + 3/(2x)	(e) 5/(2p) − 1/p	(f) 1/w − 3/(2w)

11.16.4 Simplify the following fractions:

(a) p/4 − q/12	(b) x/4 − y/2	(c) m/3 − n/9
(d) x/12 − y/6	(e) r/2 + m/10	(f) s/3 − t/15

11.16.5 Simplify the following fractions:

(a) 3x/4 − 2x/12	(b) 3x/5 − 2y/15	(c) 3m/7 + m/14
(d) 4m/(3p) − 3m/(9p)	(e) 4x/(3y) − 5x/(6y)	(f) 3r/(7s) + 2r/(14s)

Exercise 11.17 · Fractions with a lowest common multiple (Extended)

11.17.1 Simplify the following fractions:

(a) a/2 + b/3	(b) a/3 + b/5	(c) p/4 + q/7
(d) 2a/5 + b/3	(e) x/4 + 5y/9	(f) 2x/7 + 2y/5

11.17.2 Simplify the following fractions:

(a) a/2 – a/3	(b) a/3 – a/5	(c) p/4 + p/7
(d) 2a/5 + a/3	(e) x/4 + 5x/9	(f) 2x/7 + 2x/5

11.17.3 Simplify the following fractions:

(a) 3m/5 – m/2	(b) 3r/5 – r/2	(c) 5x/4 – 3x/2
(d) 2x/7 + 3x/4	(e) 11x/2 – 5x/3	(f) 2p/3 – p/2

11.17.4 Simplify the following fractions:

(a) p – p/2	(b) c – c/3	(c) x – x/5
(d) m – 2m/3	(e) q – 4q/5	(f) w – 3w/4

11.17.5 Simplify the following fractions:

(a) 2m – m/2	(b) 3m – 2m/3	(c) 2m – 5m/2
(d) 4m – 3m/2	(e) 2p – 5p/3	(f) 6q – 6q/7

11.17.6 Simplify the following fractions:

(a) p – p/r	(b) x/y + x	(c) m + m/n
(d) a/b + a	(e) 2x – x/y	(f) 2p – 3p/q

11.17.7 Simplify the following fractions:

(a) a/3 + (a + 4)/2	(b) 2b/5 + (b – 4)/3
(c) (c + 2)/4 – (2 – c)/2	(d) 2(d – 3)/7 – 3(2 – d)/2

Exercise 11.18 · Simplifying complex algebraic fractions (Extended)

11.18.1 Simplify the following algebraic fractions:

(a) $1/(x + 1) + 2/(x + 2)$	(b) $3/(m + 2) - 2/(m - 1)$	(c) $2/(p - 3) + 1/(p - 2)$
(d) $3/(w - 1) - 2/(w + 3)$	(e) $4/(y + 4) - 4/(y + 1)$	(f) $2/(m - 2) - 3/(m + 3)$

11.18.2 Simplify the following algebraic fractions:

(a) $x(x - 4)/((x - 4)(x + 2))$	(b) $y(y - 3)/((y + 3)(y - 3))$	(c) $(m + 2)(m - 2)/((m - 2)(m - 3))$
(d) $p(p + 5)/((p - 5)(p + 5))$	(e) $m(2m + 3)/((m + 4)(2m + 3))$	(f) $(m + 1)(m - 1)/((m + 2)(m - 1))$

11.18.3 Simplify the following algebraic fractions:

(a) $(x^2 - 5x)/((x + 3)(x - 5))$	(b) $(x^2 - 3x)/((x + 4)(x - 3))$	(c) $(y^2 - 7y)/((y - 7)(y - 3))$
(d) $x(x - 1)/(x^2 + 2x - 3)$	(e) $x(x + 2)/(x^2 + 4x + 4)$	(f) $x(x + 4)/(x^2 + 5x + 4)$

11.18.4 Simplify the following algebraic fractions:

(a) $(x^2 - x)/(x^2 - 1)$	(b) $(x^2 + 2x)/(x^2 + 5x + 6)$	(c) $(x^2 + 4x)/(x^2 + x - 12)$
(d) $(x^2 - 5x)/(x^2 - 3x - 10)$	(e) $(x^2 + 3x)/(x^2 - 9)$	(f) $(x^2 - 7x)/(x^2 - 49)$

Student assessment 1

SA1.1 Expand the following and simplify where possible:

(a) $5(2a - 6b + 3c)$	(b) $3x(5x - 9)$	(c) $-5y(3xy + y^2)$	(d) $3x^2(5xy + 3y^2 - x^3)$
(e) $5p - 3(2p - 4)$	(f) $4m(2m - 3) + 2(3m^2 - m)$	(g) $(1/3)(6x - 9) + (1/4)(8x + 24)$	(h) $(m/4)(6m - 8) + (m/2)(10m - 2)$

SA1.2 Factorise the following:

(a) $12a - 4b$	(b) $x^2 - 4xy$	(c) $8p^3 - 4p^2q$	(d) $24xy - 16x^2y + 8xy^2$

SA1.3 If $x = 2$, $y = -3$ and $z = 4$, evaluate the following:

(a) $2x + 3y - 4z$	(b) $10x + 2y^2 - 3z$	(c) $z^2 - y^3$
(d) $(x + y)(y - z)$	(e) $z^2 - x^2$	(f) $(z + x)(z - x)$

SA1.4 Rearrange the following formulae to make the given letter the subject:

(a) $x = 3p + q$; subject: **q**	(b) $3m - 5n = 8r$; subject: **n**	(c) $2m = 3y/t$; subject: **y**
(d) $x(w + y) = 2y$; subject: **w**	(e) $xy/(2p) = rs/t$; subject: **p**	(f) $(x + y)/w = m + n$; subject: **x**

Student assessment 2

SA2.1 Expand the following and simplify where possible:

(a) $3(2x - 3y + 5z)$	(b) $4p(2m - 7)$	(c) $-4m(2mn - n^2)$	(d) $4p^2(5pq - 2q^2 - 2p)$
(e) $4x - 2(3x + 1)$	(f) $4x(3x - 2) + 2(5x^2 - 3x)$	(g) $(1/5)(15x - 10) - (1/3)(9x - 12)$	(h) $(x/2)(4x - 6) + (x/4)(2x + 8)$

SA2.2 Factorise the following:

(a) $16p - 8q$	(b) $p^2 - 6pq$	(c) $5p^2q - 10pq^2$	(d) $9pq - 6p^2q + 12q^2p$

SA2.3 If $a = 4$, $b = 3$ and $c = -2$, evaluate the following:

(a) $3a - 2b + 3c$	(b) $5a - 3b^2$	(c) $a^2 + b^2 + c^2$
(d) $(a + b)(a - b)$	(e) $a^2 - b^2$	(f) $b^3 - c^3$

SA2.4 Rearrange the following formulae to make the given letter the subject:

(a) $p = 4m + n$; subject: **n**	(b) $4x - 3y = 5z$; subject: **y**	(c) $2x = 3y/(5p)$; subject: **y**
(d) $m(x + y) = 3w$; subject: **y**	(e) $pq/(4r) = mn/t$; subject: **r**	(f) $(p + q)/r = m - n$; subject: **q**

Student assessment 3 (Extended)

SA3.1 Factorise the following fully:

(a) $pq - 3rq + pr - 3r^2$	(b) $1 - t^4$	(c) $875^2 - 125^2$	(d) $7.5^2 - 2.5^2$

SA3.2 Expand the following and simplify where possible:

(a) $(x - 4)(x + 2)$	(b) $(x - 8)^2$	(c) $(x + y)^2$
(d) $(x - 11)(x + 11)$	(e) $(3x - 2)(2x - 3)$	(f) $(5 - 3x)^2$

SA3.3 Factorise the following:

(a) $x^2 - 4x - 77$	(b) $x^2 - 6x + 9$	(c) $x^2 - 144$
(d) $3x^2 + 3x - 18$	(e) $2x^2 + 5x - 12$	(f) $4x^2 - 20x + 25$

SA3.4 Make the given letter the subject of the formula:

(a) $mf^2 = p$; subject: **f**	(b) $m = 5t^2$; subject: **t**
(c) $A = \pi r\sqrt{(p + q)}$; subject: **p**	(d) $1/x + 1/y = 1/t$; subject: **x**

SA3.5 Simplify the following algebraic fractions:

(a) x^7/x^3	(b) $mn/p \times pq/m$	(c) $(y^3)^3/(y^2)^3$
(d) $28pq^2/(7pq^3)$	(e) $m^2n/2 \div m^2/n^2$	(f) $7b^3/c \div 4b^2/(3c^3)$

SA3.6 Simplify the following algebraic fractions:

(a) $m/11 + 3m/11 - 2m/11$	(b) $3p/8 - 9p/16$	(c) $4x/(3y) - 7x/(12y)$
(d) $3m/(15p) + 4n/(5p) - 11n/(30p)$	(e) $2(y + 4)/3 - (y - 2)$	(f) $3(y + 2) - (2y + 3)/2$

SA3.7 Simplify the following:

(a) $4/(x - 5) + 3/(x - 2)$	(b) $(a^2 - b^2)/(a + b)^2$	(c) $(x - 2)/(x^2 + x - 6)$

Student assessment 4 (Extended)

SA4.1 The volume V of a cylinder is given by the formula $V = \pi r^2 h$, where h is the height of the cylinder and r is the radius. (Calculator allowed.)

(a) Find the volume of a cylindrical post 6.5 m long and with a diameter of 20 cm. [2]

(b) Make r the subject of the formula. [1]

(c) A cylinder of height 60 cm has a volume of 5500 cm^3: find its radius correct to 3 s.f. [2]

SA4.2 The formula for the surface area of a closed cylinder is $A = 2\pi r(r + h)$, where r is the radius of the cylinder and h is its height. (Calculator allowed.)

(a) Find the surface area of a cylinder of radius 12 cm and height 20 cm, giving your answer to 3 s.f. [2]

(b) Rearrange the formula to make h the subject. [2]

(c) What is the height of a cylinder of surface area 500 cm^2 and radius 5 cm? Give your answer to 3 s.f. [2]

SA4.3 The formula for finding the length d of the body diagonal of a cuboid whose dimensions are x, y and z is $d = \sqrt{(x^2 + y^2 + z^2)}$. (Calculator allowed.)

(a) Find d when x = 2, y = 3 and z = 4. [1]

(b) How long is the body diagonal of a block of concrete in the shape of a rectangular prism of dimensions 2 m, 3 m and 75 cm? [2]

(c) Rearrange the formula to make x the subject. [2]

(d) Find x when d = 0.86, y = 0.25 and z = 0.41. [1]

SA4.4 A pendulum of length l metres takes T seconds to complete one full oscillation. The formula for T is $T = 2\pi\sqrt{(l/g)}$, where g m/s^2 is the acceleration due to gravity. (Calculator allowed.)

(a) Find T if l = 5 and g = 10. [1]

(b) Rearrange the formula to make l the subject of the formula. [2]

(c) How long is a pendulum which takes 3 seconds for one oscillation, if g = 10? [2]

Check yourself

1. Draw a grid and use it to expand $(3x - 2)(x + 5)$. Explain where each of the four terms comes from. [3]

2. A student writes $(x + 4)^2 = x^2 + 16$. Explain the mistake and give the correct expansion. [2]

3. Explain why $4x^2 - 9$ can be factorised, but $4x^2 + 9$ cannot be factorised as a difference of two squares. [2]

4. Make b the subject of $a = (b + c)/d$. Write down what you did to both sides at each step. [3]

5. Explain why $x/3 + x/4$ is not equal to $2x/7$, and simplify it correctly. [2]

6. Factorise $x^2 - x - 20$, then use your answer to simplify $(x^2 - x - 20)/(x^2 - 25)$. [3]

Chapter quiz

Q1 Expand $3(2x - 5)$.

A	B	C	D
$6x - 5$	$6x - 15$	$5x - 15$	$6x + 15$

Q2 Expand and simplify $4x - 2(x - 3)$.

A	B	C	D
$2x - 6$	$2x + 6$	$2x - 3$	$6x - 6$

Q3 Expand and simplify $(x + 4)(x - 2)$.

A	B	C	D
$x^2 - 8$	$x^2 + 6x - 8$	$x^2 + 2x - 8$	$x^2 - 2x - 8$

Q4 Expand $(x - 5)^2$.

A	B	C	D
$x^2 - 25$	$x^2 + 25$	$x^2 - 10x - 25$	$x^2 - 10x + 25$

Q5 Factorise fully $12ab - 8a^2$.

A	B	C	D
$2a(6b - 4a)$	$4a(3b - 2a)$	$4(3ab - 2a^2)$	$a(12b - 8a)$

Q6 Factorise $9x^2 - 25$.

A	B	C	D
$(3x - 5)^2$	$(9x + 5)(x - 5)$	$(3x + 5)(3x - 5)$	$(3x + 25)(3x - 1)$

Q7 Factorise $x^2 - 3x - 10$.

A	B	C	D
$(x - 5)(x + 2)$	$(x + 5)(x - 2)$	$(x - 10)(x + 1)$	$(x - 5)(x - 2)$

Q8 By factorising, work out $98^2 - 2^2$.

A	B	C	D
9600	9604	9400	9996

Q9 Find $a^2 - b^3$ when $a = 3$ and $b = -2$.

A	B	C	D
1	−1	17	5

Q10 Make x the subject of $y = 3x - 5$.

A	B	C	D
$x = 3y - 5$	$x = (y - 5)/3$	$x = (y + 5)/3$	$x = 3(y + 5)$

Q11 Make r the subject of $A = \pi r^2$, where $r > 0$.

A	B	C	D
$r = A/\pi$	$r = \sqrt{A}/\pi$	$r = \sqrt{(A/\pi)}$	$r = A/(2\pi)$

Q12 Simplify $x/3 + x/4$.

A	B	C	D
$2x/7$	$7x/12$	$x/7$	$7x/7$

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-11-algebraic-representation-and-manipulation/